A. **Path Derivative Rules.** Derivatives come equipped with rules.

Sum:	$\left(\mathbf{r}_1 + \mathbf{r}_2\right)' = \mathbf{r}_1' + \mathbf{r}_2'$
Constant Multiple:	$\left(c\,\mathbf{r}\right)' = c\,\mathbf{r}'$
Dot Product:	$\left(\mathbf{r}_1 \cdot \mathbf{r}_2\right)' = \left(\mathbf{r}_1' \cdot \mathbf{r}_2\right) + \left(\mathbf{r}_1 \cdot \mathbf{r}_2'\right)$
Cross Product:	$\left(\mathbf{r}_1 \times \mathbf{r}_2\right)' = \left(\mathbf{r}_1' \times \mathbf{r}_2\right) + \left(\mathbf{r}_1 \times \mathbf{r}_2'\right)$

$\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$ denote differentiable paths, and c denotes a constant scalar. We write $\mathbf{r}_1$ and $\mathbf{r}_2$ without the (t) for ease of readibility.

Remember that for cross products, order matters! Changing order flips sign. Therefore the order in the cross product rule is important to pay attention to!

Example 1. Let $\mathbf{r}(t)$ be a differentiable parametrization for a curve that lies entirely on the sphere:

$$x^2 + y^2 + z^2 = 1$$

Show that $\mathbf{r}'(t)$ is always orthogonal to $\mathbf{r}(t)$.

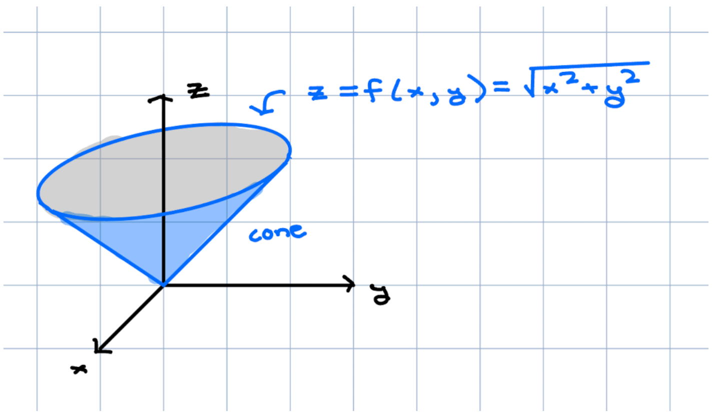
These vectors will be orthogonal when $\mathbf{r}'(t) \cdot \mathbf{r}(t) = 0$. Does the lefthand side not look reminiscent of a piece of the dot product rule?

B. **Multivariable Functions and Level Curves.** The functions we encounter in this course will generally involve multiple input variables. For example:

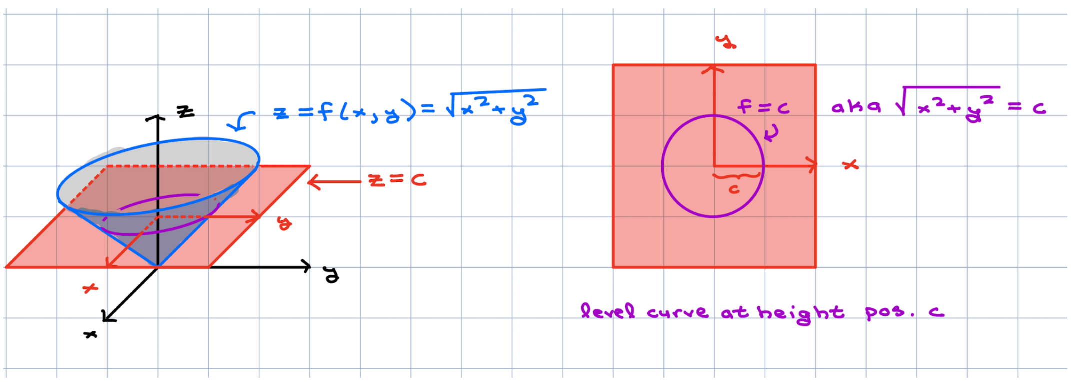
$$f(x, y) = \sqrt{x^2 + y^2}$$

To graph a function, we set the outputs equal to another variable. In this case, the graph is defined by:

$$z = \sqrt{x^2 + y^2}$$

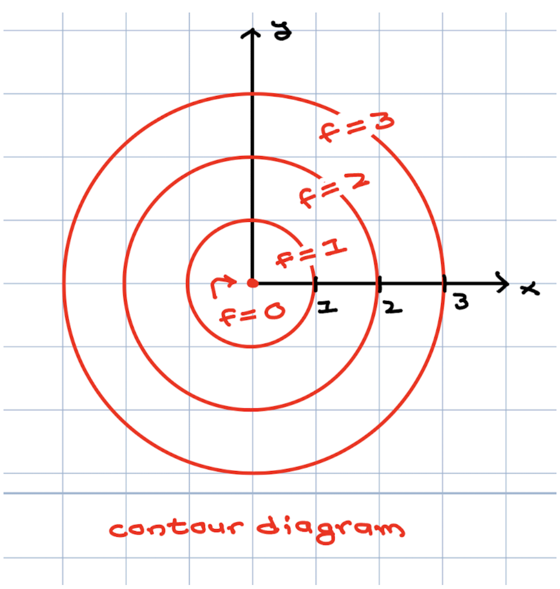


A **level set** at height c is the set of inputs with output c:



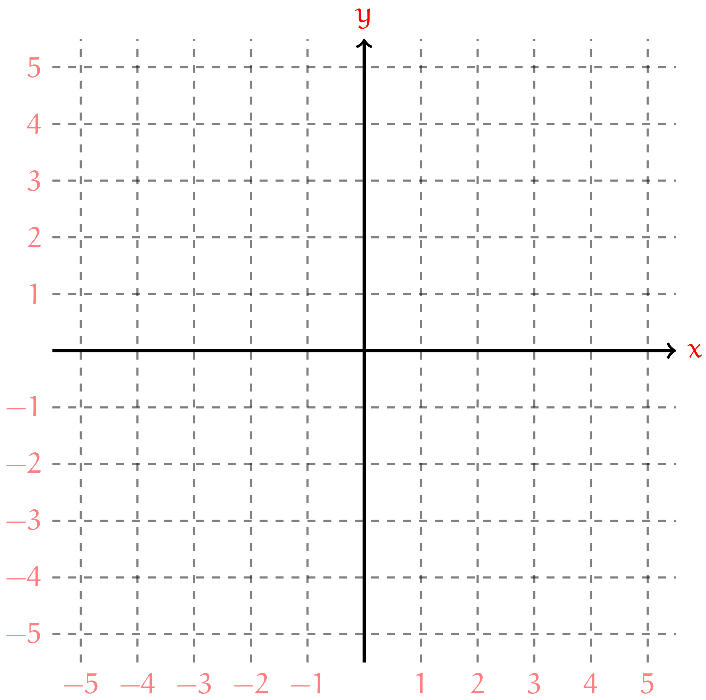
In this case, the level set is a curve, and therefore we would call it a **level curve**. Another name for a level curve is a **contour**.

If we arrange many level curves together we get a **contour diagram**:



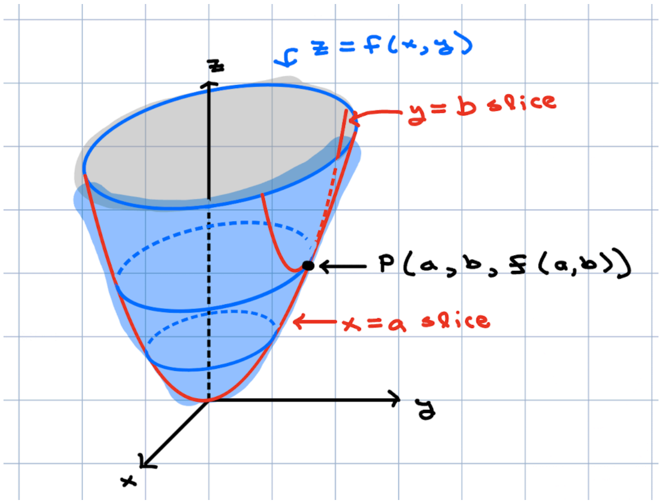
Example 2. Sketch a contour diagram for:

$$f(x, y) = y - 2x$$



The graph of this function is defined by the equation $z = y - 2x$. This is the equation of a plane!

C. **Partial Derivatives.** Consider the function $z = f(x, y)$ and the point $P(a, b, f(a, b))$.



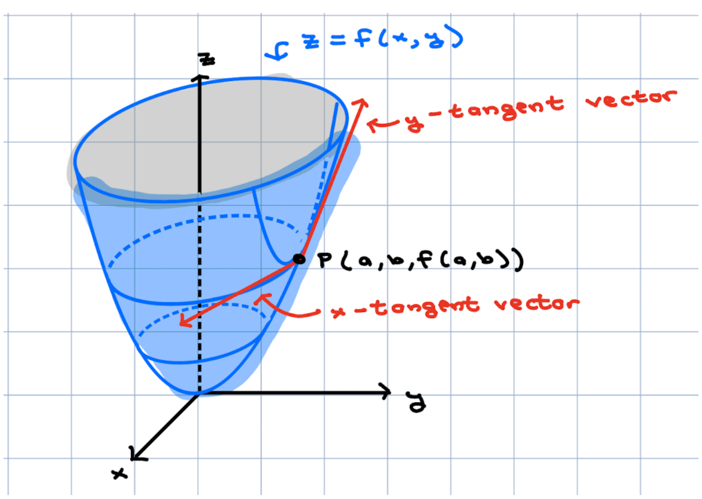
We parametrize the $y = b$ and $x = a$ slices using x and y as a parameters:

$\mathbf{r}_{y=b}(x) =$

$\mathbf{r}_{x=a}(y) =$

x–tangent at (a, b) is: $(\mathbf{r}_{y=b})'(a) =$

y–tangent at (a, b) is: $(\mathbf{r}_{x=a})'(b) =$



The x**–partial derivative** of f at (a, b) is the vertical component of the x–tangent vector at P and is denoted $f_x(a, b)$. It tells you the rate of change of f with respect to x.

The y**–partial derivative** of f at (a, b) is the vertical component of the y–tangent vector at P and is denoted $f_y(a, b)$. It tells you the rate of change of f with respect to y.

Example 3. Let $f(x, y) = ye^{2xy}$ and find:

$f_x(0, 1) =$

$f_y(0, 1) =$

$$\frac{\partial f}{\partial x} = f_x(x, y) =$$

$$\frac{\partial f}{\partial y} = f_y(x, y) =$$

To compute a x-partial derivative at (a, b), hold y constant at $y = b$, and then take a derivative with respect to x, and lastly plug in $x = a$.

To compute a y-partial derivative at (a, b), hold x constant at $x = a$, and then take a derivative with respect to y, and lastly plug in $y = b$.

The notation $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ is analagous to the familiar $\frac{df}{dx}$ of 1D calculus, but with ∂ instead of d to emphasize that this is a partial derivative. This notation is nice as it emphasizes that partial derivatives are ratios of changes in f over changes in either x or y, since the notation $\partial\star/\partial\star$ denotes an infinitesimal change in $\star$.

To compute $\frac{\partial f}{\partial x}$ you should treat y as a constant and take a derivative with respect to x.

To compute $\frac{\partial f}{\partial y}$ you should treat x as a constant and take a derivative with respect to y.