Example 1. Let E be the solid cylinder $x^2 + y^2 \leqslant 9$ and $0 \leqslant z \leqslant 4$. Let $\mathcal{S}$ be the boundary of E, oriented with **inward** normals. If:

$$\mathbf{F} = \left\langle e^{y^2 z^2} \,,\; y + \sin(e^{x+z}) \,,\; z^2 - z \right\rangle$$

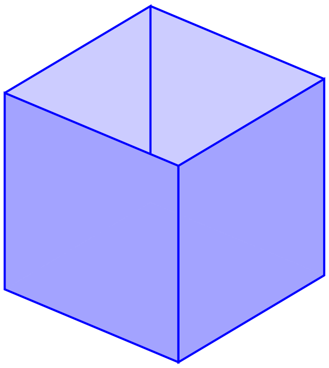
then find $\displaystyle\oiint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S}$

Remember that if you want to compute a triple integral with cylindrical coordinates, then $dV = r\ dz dr d\theta$.

Example 2. Let $\mathcal{S}$ be the surface of the box:

$$[-1, 1] \times [-1, 1] \times [0, 2]$$

but without its top, and oriented with **outward** normals.



Use the divergence theorem to find the flux of $\mathbf{F}$ across $\mathcal{S}$ where:

$$\mathbf{F} = (2x + y + z)\mathbf{i} + (x + 2y - z)\mathbf{j} + (x + y + 2z)\mathbf{k}$$

This surface is not closed, as the paper–cut boundary of the surface is given by the top edges. This is an issue, because the divergence theorem only applies to closed surfaces. Devastating. One strategy in this scenario is to **close off** the surface, by introducing a new surface $\mathcal{S}'$ that, combined with $\mathcal{S}$, forms a closed surface. In our case we will simply include the missing top–face $\mathcal{S}'$. Since $\mathcal{S}$ is oriented with outward normals, we also orient $\mathcal{S}'$ with outward normals. For the top–face, outwards means upwards!

Remember that for a graph $z = f(x, y)$ parametrized by x and y we have:

$$\mathbf{dS} = \pm\langle -f_x, -f_y, 1\rangle \; dxdy$$