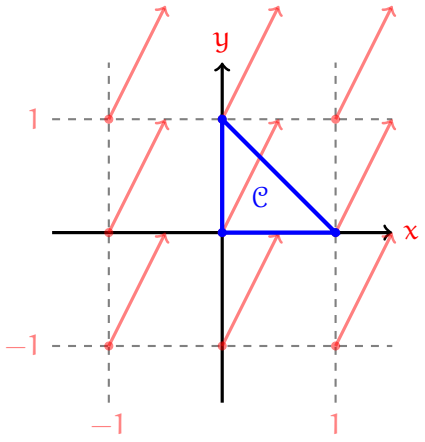
Example 1. Integrate

$\vec{F}(x, y) = \frac{1}{2}\langle 1, 2\rangle$

over the **clockwise** oriented triangle $\mathcal{C}$ depicted below.



Solution. Our curve is composed of three segments.

$$\int_{\mathcal{C}} \vec{F} \cdot d\vec{r} =$$

We first address the **horizontal** segment. It is parametrized right–to–left as

$\vec{r}(t) =$

$d\vec{r} =$

$$\int_{\vec{r}} \vec{F} \cdot d\vec{r} =$$

Next we address the **vertical** segment. It is parametrized bottom–to–top as

$\vec{r}(t) =$

$d\vec{r} =$

$$\int_{\vec{r}} \vec{F} \cdot d\vec{r} =$$

Lastly we address the **diagonal** segment. It is parametrized left–to–right as

$\vec{r}(t) =$

$d\vec{r} =$

$$\int_{\vec{r}} \vec{F} \cdot d\vec{r} =$$

For our final answer, we sum to get

$$\int_{\mathcal{C}} \vec{F} \cdot d\vec{r} =$$

A. **Alternative Notation.** You may see different notation for vector line integrals.

We write

$\vec{F} =$

$d\vec{r} =$

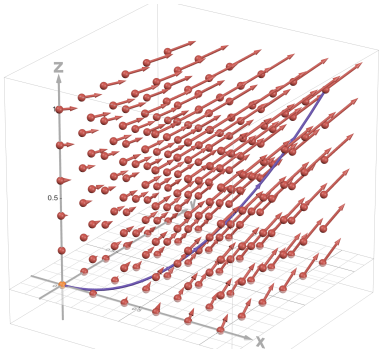
in which case

$\vec{F} \cdot d\vec{r} =$

Example 2. Find

$$\int_{\vec{r}} (y+z)\,dx + (x+z)\,dy + (x+y)\,dz$$

where $\vec{r}(t) = \left(t, t^2, t^3\right)$ and $0 \leqslant t \leqslant 1$.



Solution. This is alternative notation for the vector line integral of

$\vec{F}(x, y, z) =$

along the curve parametrized for $0 \leqslant t \leqslant 1$ by

$x =$

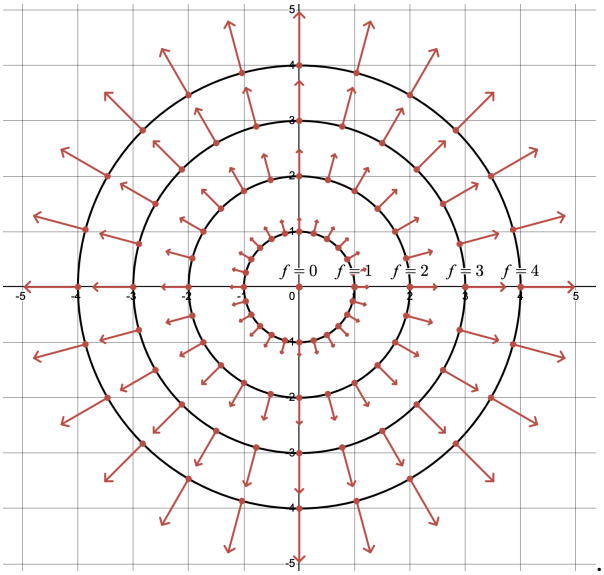
$y =$

$z =$

We now rewrite the integral entirely in terms of the parameter t.

$$\int_{\vec{r}} (y+z)\,dx + (x+z)\,dy + (x+y)\,dz$$

B. **Conservative Vector Fields.** We examine gradient vector fields.



Vector field $\vec{F}$ is **conservative** if it is a gradient vector field, in other words if

$\vec{F} =$

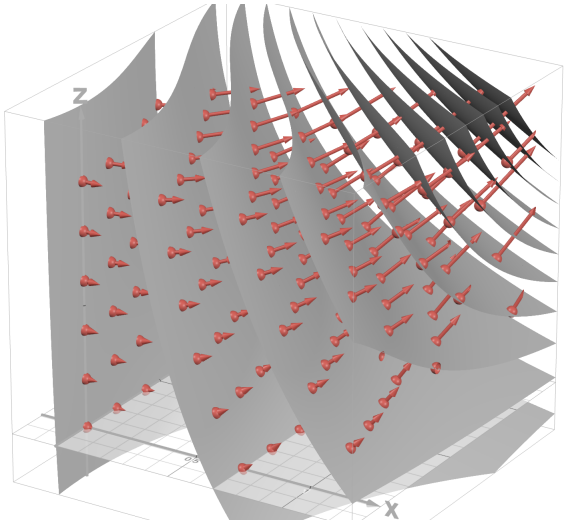
for a function f that we call a **potential** for $\vec{F}$.

If a conservative vector field represents fluid flow, fluid always flows

and thus a conservative vector field never exhibits any:

Example 3. Show the vector field is conservative by finding all potentials.

$\vec{F}(x,y,z) = (e^{yz}+z)\vec{\imath} + (xze^{yz}+2yz)\vec{\jmath} + (xye^{yz}+y^2+x)\vec{k}$



Solution. Finding a potential f is equivalent to solving the following system.

$\boxed{1}$ $f_x(x,y,z) =$

$\boxed{2}$ $f_y(x,y,z) =$

$\boxed{3}$ $f_z(x,y,z) =$

First we integrate $\boxed{1}$ with respect to x. We note that a constant dx may involve:

$\int \boxed{1}\, dx$

We substitute this expression for $f(x,y,z)$ into $\boxed{2}$ and solve for $g(y,z)$.

$\boxed{2}$

At this point we have established

$f(x,y,z) =$

We substitute this expression for $f(x,y,z)$ into $\boxed{3}$ and solve for $h(z)$.

$\boxed{3}$

In conclusion, the general form of a potential f for $\vec{F}$ is given by

$f(x,y,z) =$