A. **Fundamental Theorem of Line Integrals.** Let $\vec{F} = \vec{\nabla} f$ be conservative.

Say we wish to integrate $\vec{F}$ over a curve, parametrized by

This would entail computing

$$\int_{\vec{r}} \vec{F} \cdot d\vec{r} =$$

But the chain rule for functions of paths $f(\vec{r}(t))$ stated that

$$\frac{df}{dt} =$$

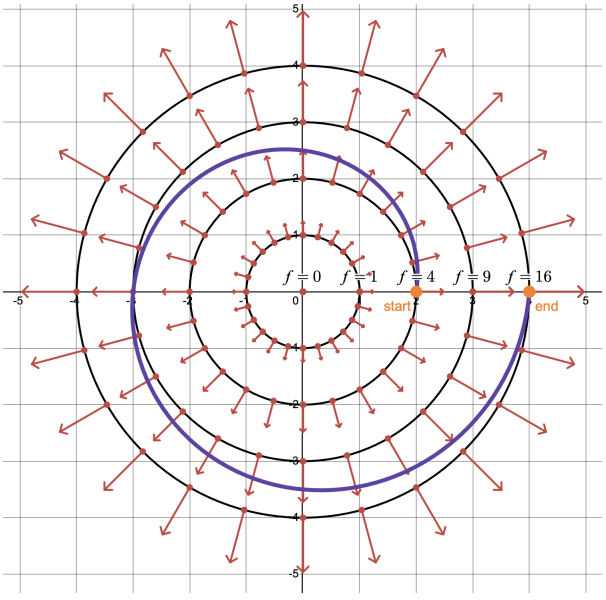
and thus we may continue

$$\cdots =$$

Fundamental Theorem of Line Integrals. Let $\vec{F}$ be conservative on the path $\vec{r}(t)$ with $a \leqslant t \leqslant b$

and let f be a potential of $\vec{F}$. Then

$$\int_{\vec{r}} \vec{F} \cdot d\vec{r} =$$



Example 1. You found last class that the vector field

$\vec{F}(x,y,z) = (e^{yz}+z)\vec{\imath} + (xze^{yz}+2yz)\vec{\jmath} + (xye^{yz}+y^2+x)\vec{k}$

is conservative, and has potential function

$f(x,y,z) = xe^{yz} + y^2z + xz$

Use this to compute $\int_{\vec{r}} \vec{F}\cdot d\vec{r}$ for the path $\vec{r}(t) = \langle t, t^2, t^3\rangle$ with $0 \leqslant t \leqslant 1$.

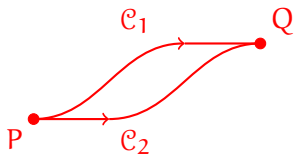
Solution. By the fundamental theorem of line integral we calculate

[start] =

[end] =

$\int_{\vec{r}} \vec{F}\cdot d\vec{r} =$

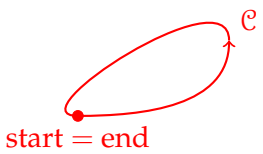
B. **Path Independence.** Consider two paths with the same start and end.



Path Independence. Let $\vec{F}$ be conservative on paths C_1 and C_2 sharing

Then:

As a special case consider a **closed curve**, i.e. a curve with:



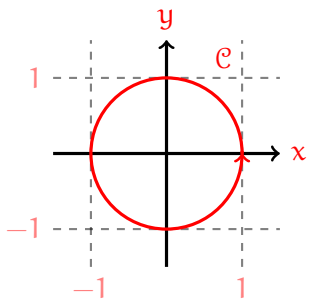
Closed Curves. Let $\vec{F}$ be conservative on closed curve C. Then

where $\oint$ is just notation reminding us the curve is closed.

Example 2. Consider the vector field

$$\vec{F}(x,y) = \frac{-y\vec{\imath} + x\vec{\jmath}}{x^2 + y^2}$$

Show $\vec{F}$ is not conservative by integrating over the counterclockwise unit circle $\mathcal{C}$.



Solution. Observe that $\mathcal{C}$ is a:

Therefore, if we find that

then $\vec{F}$ cannot be:

Let us calculate this integral. First we parametrize $\mathcal{C}$ counterclockwise.

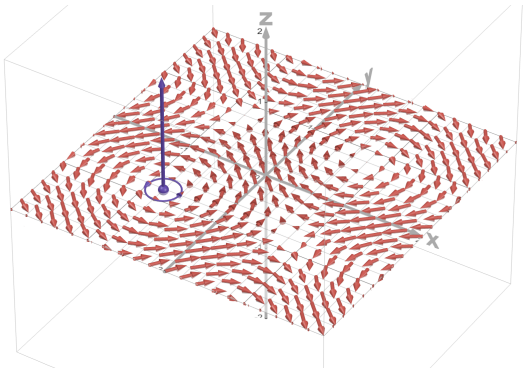
Next we write the integral in terms of our parameter.

$$\int_{\mathcal{C}} \vec{F} \cdot d\vec{r} =$$

C. **Curl.** We introduce introduce a way to assess "circulation" in a vector field.

Let $\vec{F}$ be a vector field and P be a point.

The **curl** of $\vec{F}$ at P is a vector denoted:



- the direction of the curl is determined by the:
- the curl vector is longer where:

Curl is calculated using the **operator**

$\vec{\nabla} =$

according to the rule

$\operatorname{curl} \vec{F} =$

For example we compute $\operatorname{curl} \vec{F}(x, y, z)$ for

$\vec{F}(x, y, z) = \langle xy \sin z, y - xe^z, xyz \rangle$