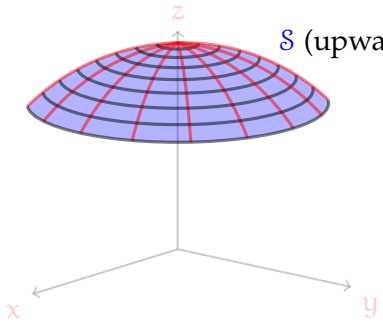
Example 1. Let us revisit the example from last time. Let

$$\vec{F} = -y\vec{\imath} + \left[x + (z-1)xe^x\right]\vec{\jmath} + \left[x^2 + y^2\right]\vec{k}$$

and S be the spherical cap $x^2 + y^2 + z^2 = 2$ with $z \geqslant 1$ and oriented upwards.

Our goal is to find $\iint_S \operatorname{curl} \vec{F} \cdot d\vec{S}$.



Let us pick a better surface S' with the same oriented boundary as S. We choose

Stokes's says integrating curl over a surface yields circulation on the boundary.

Since S and S' share a common oriented boundary this implies

So we may work with the nicer surface S' for this problem.

Using shortcut for upwards $d\vec{S} = \langle -\frac{\partial z}{\partial x}, -\frac{\partial z}{\partial y}, 1\rangle \, dA$ for S' we obtain

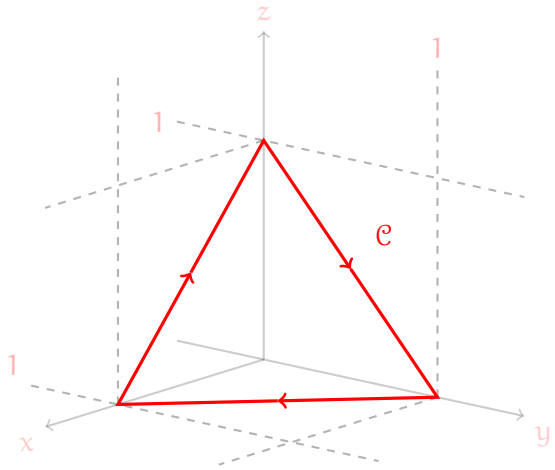
Next we simplify the vector field on our surface using $z = 1$ and compute curl.

and now we return to integrating the curl over S'.

$$\iint_{S'} \operatorname{curl} \vec{F} \cdot d\vec{S} =$$

Example 2. Let $\mathcal{C}$ be the oriented curve of straight line segments

$(1,0,0) \to (0,0,1) \to (0,1,0) \to (1,0,0)$



Use Stokes's Theorem to find $\oint_{\mathcal{C}} \left(x\sin(e^x) - xz\right)\, dx - xy\, dy + (z^2 + y)\, dz$.

Solution. Observe that $\mathcal{C}$ is the boundary of the surface $\mathcal{S}$ defined by

We set up Stokes's with vector field $\vec{F} = \left\langle x\sin(e^x) - xz, -xy, z^2 + y \right\rangle$.

Another way to think about the orientations for Stokes's Theorem is the:

The given orientation on $\mathcal{C}$ corresponds to the orientation on $\mathcal{S}$ that is:

Adjusting shortcut for <u>upwards</u> $d\vec{S} = \langle -\frac{\partial z}{\partial x}, -\frac{\partial z}{\partial y}, 1\rangle\, dA$ for our $\mathcal{S}'$ we obtain

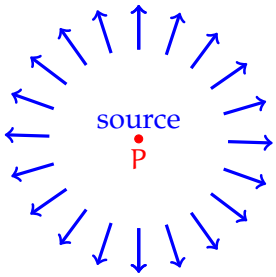
Next we compute the curl.

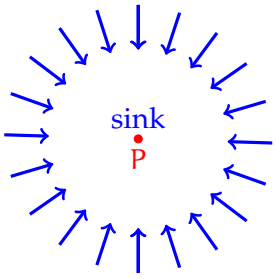
And lastly we integrate the curl over our surface.

$\iint_{\mathcal{S}} \operatorname{curl} \vec{F} \cdot d\vec{S} =$

A. **Divergence.** Let vector field $\vec{F}$ represent fluid flow.

The **divergence** of $\vec{F}$ at point P measures the net fluid flow rate away/towards P.





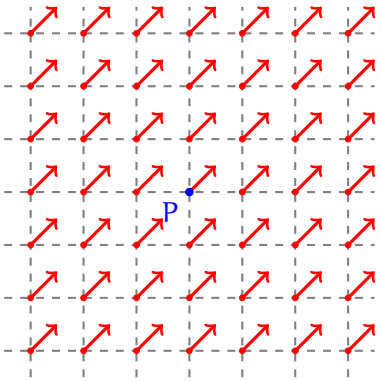
Divergence is calculated using the operator $\vec{\nabla} = \langle \partial_x, \partial_y, \partial_z \rangle$ by the formula:

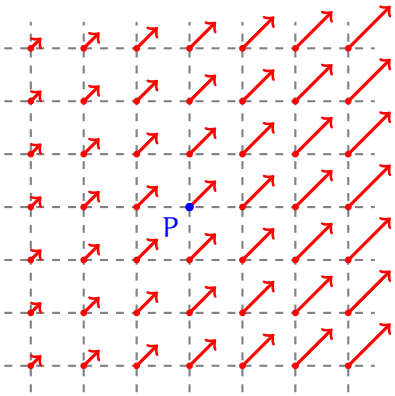
$\operatorname{div} \vec{F} =$

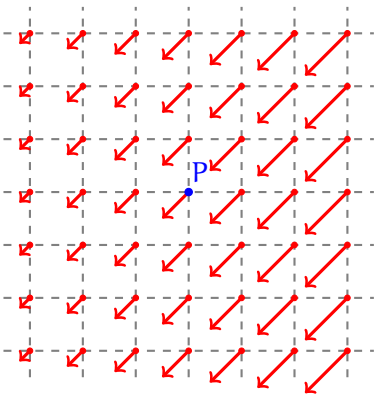
For example if $\vec{F} = (xy \sin z + y)\vec{\imath} + (y - xz^2)\vec{\jmath} + (xyz)\vec{k}$ then

$\operatorname{div} \vec{F} =$

Example 3. For each $\vec{F}$, decide whether $\operatorname{div} \vec{F}(P)$ is zero, positive, or negative.







Solution. We compare net flow away–from (+) versus towards (−) the point P.

Example 4. Let $\vec{F}$ be a twice–continuously differentiable vector field and show

$\operatorname{div}\operatorname{curl}\vec{F} = 0$

Solution. We write $\vec{F} = \langle P, Q, R\rangle$ and compute

$\operatorname{curl}\vec{F} =$

And then take the divergence

$\operatorname{div}\operatorname{curl}\vec{F} =$