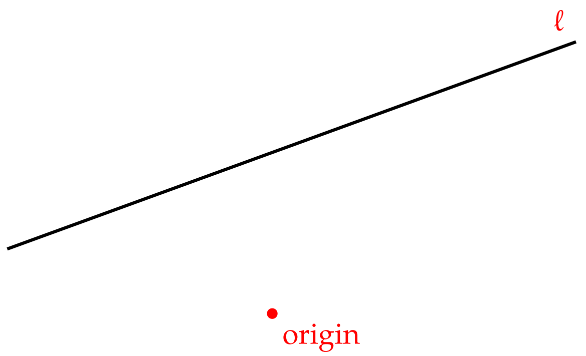
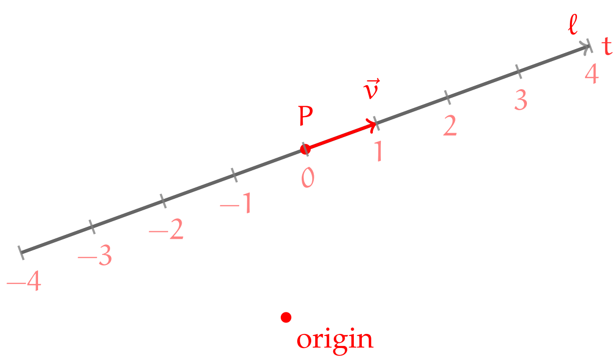
A. **Lines.** Let us use vectors to describe lines.

For us lines are straight and continue indefinitely in both directions. We will find them in the xy-plane or in xyz-space.



We will provide a method to **locate** points on the line using **scalars** t. We begin by selecting a point P on the line as a **start** and a vector $\vec{v}$ parallel to the line as a **direction**.

We call a nonzero vector that is parallel to the line a **direction vector**.



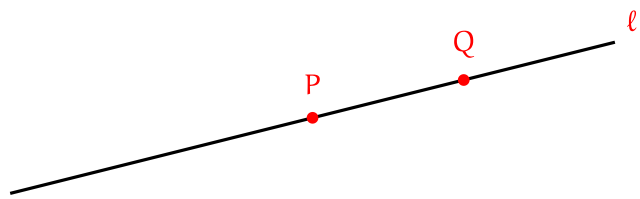
The position vector for the point at coordinate t along this line is:

$\vec{r}(t) =$

The function $\vec{r}(t)$ is called a **parametrization** and t is called the **parameter**.

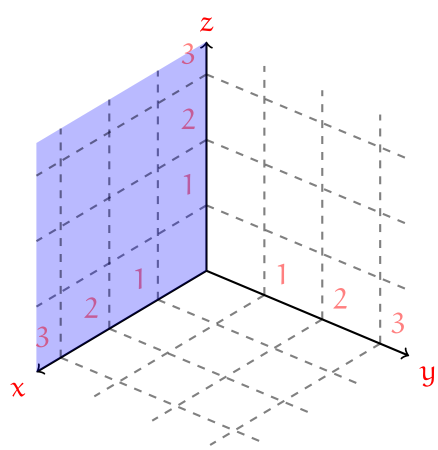
A classic interpretation of a parametrization is that it gives the **position** $\vec{r}(t)$ of a **particle** moving along the line at time t. We use Desmos to visualize this.

Example 1. Consider the line ℓ through the points $P(0,4,2)$ and $Q(-3,2,-7)$.



(a) Parametrize the line.

(b) Find the point where this line intersects the xz–plane.



The xy-plane is defined by the equation $z = 0$, the xz-plane by the equation $y = 0$, and the yz-plane by $x = 0$.

Example 2. Are the lines with the following parametrizations parallel?

$$\ell_1 : \vec{r}_1(t) = \langle 8 - 8t, 4 - 3t, 2 + t \rangle$$

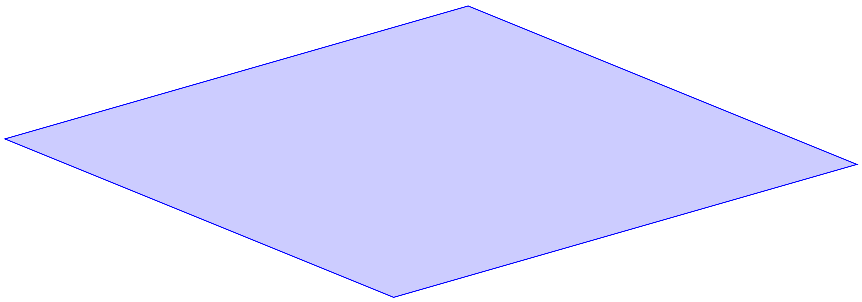
$$\ell_2 : \vec{r}_2(s) = \langle 4s, 1 + 1.5s, 8 - 0.5s \rangle$$

Lines are said to be parallel if they have parallel direction vectors. And vectors are parallel if one is a scalar multiple of the other!

We are using the idea that these parametrizations have the form: (point) + (parameter)(direction vector) to quickly locate our direction vectors.

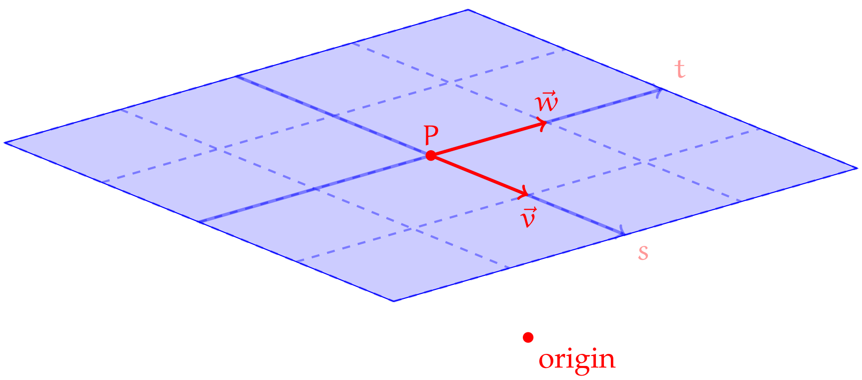
B. **Planes.** The 2D version of a line is a plane.

Planes are flat like pieces of paper, but they continue indefinitely in all directions! We will typically consider them living inside cozy xyz-space



First we select a **start** point P on the plane, and two **directions** $\vec{v}$ and $\vec{w}$ that are parallel to the plane but not to each other. Then the parameter s will measure units of $\vec{v}$, and the parameter t will measure units of $\vec{w}$.

If we pick two vectors that are parallel to each other, well we will not be able to use the following strategy to obtain every point on the plane. We will be confined to points on a particular line in the plane. Why would I want to be on a singular line when I could be on a lines worth of lines, aka a plane?



The position vector for the point at coordinate (s, t) along this plane is:

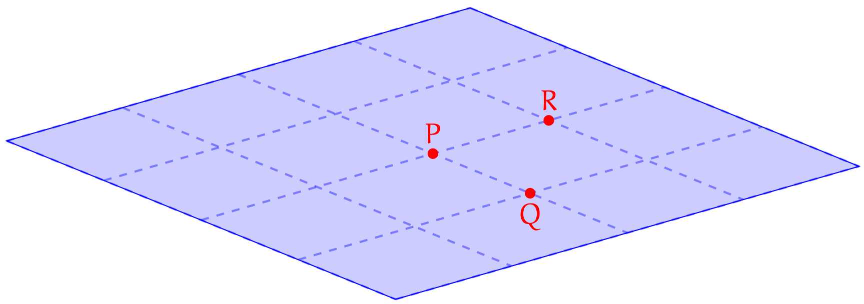
$$\vec{X}(s,t) =$$

The function $\vec{X}(s,t)$ is called a **parametrization** for the plane and s and t are called the **parameters**.

Just as with lines, for a given plane we have multiple choices of parametrizations. We could easily obtain different parametrizations by selecting different points and different vectors.

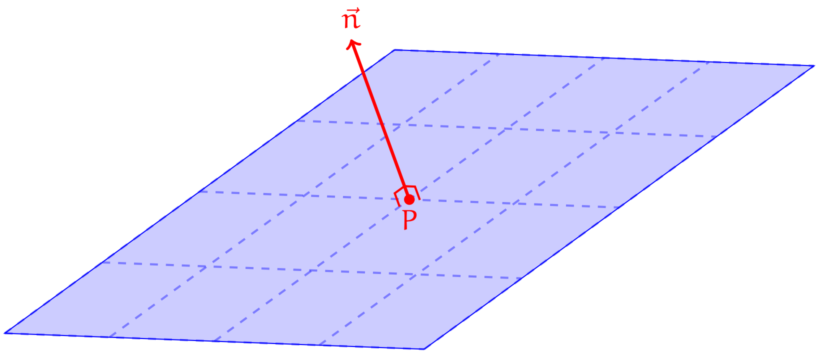
We can use Desmos to visualize the parameterization of a plane.

Example 3. Parametrize the plane that passes through the points $P(1,-2,1)$, $Q(-4,5,-8)$, and $R(7,-8,7)$.



As before this sketch was made without a thought to correctness of orientation an perspective. I nonetheless hope against hope that it will still be of use. Spoiler: it is. The main way it could have failed us is if the three of P, Q, and R were on a singular line. But of course, then they would not determine a plane.

C. **Normal Vectors.** There is an alternative, effective method for describing planes in 3D space. We still select a point P on the plane, but instead of selecting vectors parallel to the plane, we select a nonzero vector $\vec{n}$ that is **orthogonal** to the plane. We call such a vector $\vec{n}$ a **normal vector** to the plane.



If **another** point X is on this plane, then:

Given a **point** P on a plane and a **normal** vector $\vec{n}$ to the plane, you can decide whether a point $X = (x, y, z)$ is also on the plane by checking the equation:

which we call the **scalar equation** of the plane.

We think of $\vec{x} = \langle x, y, z \rangle$ as a vector of variables, and a solution to the equation as a selection of those variables that satisfies the equation. To show that you really understand this idea: you know that the equation $2x + y = 1$ defines a line in xy-plane. How does it define a line? It gives you a rule for assessing whether a point (x, y) is on the line. For example $(0, 1)$ is on the line because $2(0) + (1) = 1$ satisfies the equation. But $(1, 1)$ is not on the line because $2(1) + 1 \neq 1$ does not satisfy the equation.

For example, if a plane has normal $\vec{n} = \langle 1, 2, 3 \rangle$ and contains point $P(1, 1, 1)$, then its **scalar equation** is:

We can decide whether the following points are **also** on the plane by plugging them into the scalar equation:

$Q = (3, 2, 1)$

$R = (2, 2, 0)$

When multiplied out, the scalar equation of a plane has form:

where a normal is given by $\vec{n} =$