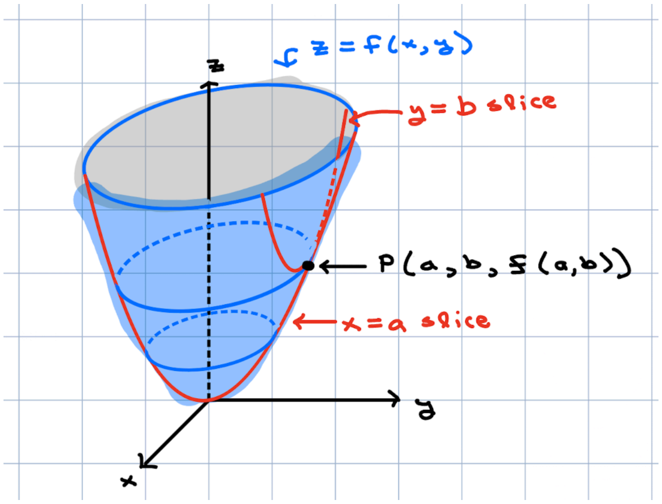
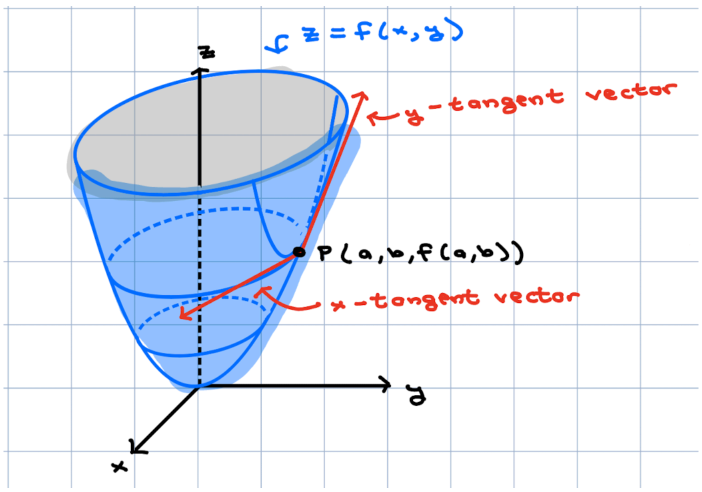
A. **Partial Derivatives.** Consider the graph $z = f(x, y)$ and the point $P(a, b, f(a, b))$.



At the point P, we can consider the slope of the surface in the y–direction, and the slope of the surface in the x–direction.



Partial Derivatives. Consider surface $z = f(x, y)$ and point $P(a, b, f(a, b))$.

The x**–partial derivative** of f at (a, b) is:

$f_x(a, b) =$

and measures the slope of the surface at P in the x–direction, i.e. the rate of change of f from (a, b) as x increases.

The y**–partial derivative** of f at (a, b) is:

$f_y(a, b) =$

and measures the slope of the surface at P in the y–direction, i.e. the rate of change of f from (a, b) as y increases.

Example 1. Let $f(x,y) = ye^{2xy}$ and find:

$f_x(0,1) =$

$f_y(0,1) =$

$$\frac{\partial f}{\partial x} = f_x(x,y) =$$

$$\frac{\partial f}{\partial y} = f_y(x,y) =$$

To compute a x-partial derivative at (a,b), hold y constant at $y = b$, and then take a derivative with respect to x, and lastly plug in $x = a$.

To compute a y-partial derivative at (a,b), hold x constant at $x = a$, and then take a derivative with respect to y, and lastly plug in $y = b$.

The notation $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ is analagous to the familiar $\frac{df}{dx}$ of 1D calculus, but with ∂ instead of d to emphasize that this is a partial derivative. This notation is nice as it emphasizes that partial derivatives are ratios of changes in f over changes in either x or y, since the notation $\partial\star/\partial\star$ denotes an infinitesimal change in $\star$.

To compute $\frac{\partial f}{\partial x}$ you should treat y as a constant and take a derivative with respect to x.

To compute $\frac{\partial f}{\partial y}$ you should treat x as a constant and take a derivative with respect to y.

Example 2. Let $f(x, y, z) = y^2 \sin(x) + e^z$ and find:

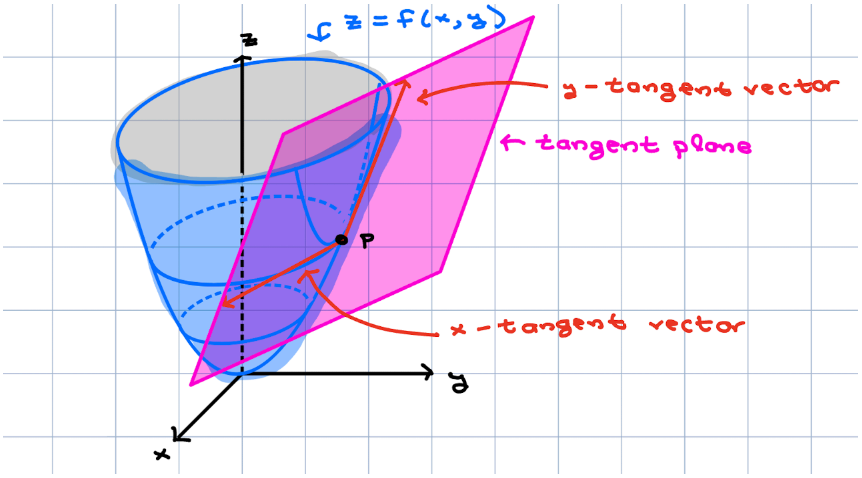
Even when there are more variables, to calculate $\frac{\partial f}{\partial \star}$ you should treat all other variables as constants and take a derivative with respect to $\star$.

$$\frac{\partial f}{\partial x} = f_x(x, y, z) =$$

$$\frac{\partial f}{\partial y} = f_y(x, y, z) =$$

$$\frac{\partial f}{\partial z} = f_z(x, y, z) =$$

B. **Tangent Planes.** The tangent vectors to the surface $z = f(x, y)$ at $P(a, b, f(a, b))$ together form a plane called the **tangent plane**:



The tangent vectors depicted above are:

$\left[x\text{–tangent at } (a, b)\right] =$

$\left[y\text{–tangent at } (a, b)\right] =$

The **tangent plane** to the $z = f(x, y)$ at $P(a, b, f(a, b))$ has normal vector:

$\vec{n} =$

and therefore, a scalar equation for this tangent plane is given by:

When we zoom in to the point of tangency of the tangent plane, we see that the tangent plane is a good approximation for the surface, at least for x and y near $x = a$ and $y = b$.

The **linear approximation** for $f(x, y)$ that is suitable near $x = a$ and $y = b$ is:

$L(x, y) =$

We use the word **linear** because the tangent plane is defined by a linear equation. And this linear approximation is literally the same as the tangent plane, or more precisely, the graph of the linear approximation is the tangent plane. In other words, you find the linear approximation by solving for z in the equation of the tangent plane.

Example 3. Both parts relate to the function $f(x, y) = ye^{2xy}$.

(a) Find an equation for the tangent plane to $z = f(x, y)$ at the point $(0, 1, 1)$.

For this exact function we calculated $f_x(0, 1) = 2$ and $f_y(0, 1) = 1$ in an earlier example.

(b) Use part a and a linear approximation to find a rational number estimate for:

$0.9e^{0.18}$

A rational number is number that equals a fraction of integers. An integer is in the infinite list $0, \pm 1, \pm 2, \ldots$