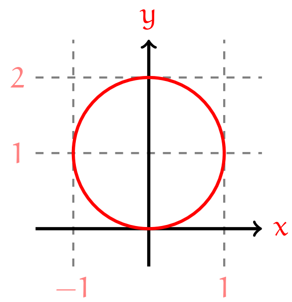
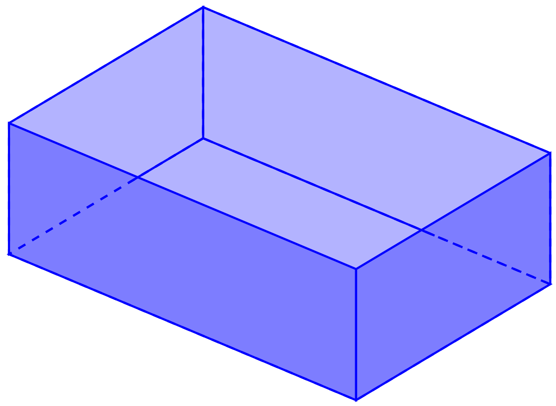
Example 1. Find all solutions to the Lagrange equation for:

$f(x, y) = 2x^2 + \frac{y^3}{3} - y$ subject to $x^2 + (y-1)^2 = 1$.



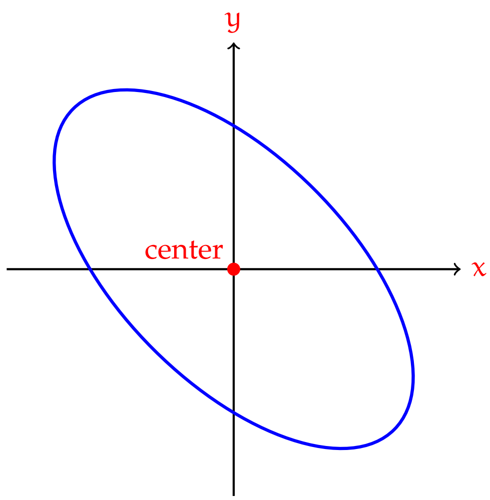
Example 2. A rectangular box with no lid is made using 48 cm^2 of cardboard. Find the box dimensions that maximize volume.



The extreme value theorem does not apply here, because the feasible set is unbounded. Well. . . if you include the restrictions that all edges must have nonnegative length, and each face must have area $\leqslant 48$, then actually the feasible set does become compact. So the extreme value theorem does apply. But this is all getting a little too technical for us.

Example 3. Find the circle of largest radius centered at the origin that can be inscribed in the ellipse:

$$x^2 + xy + y^2 = 3$$



In order for a circle centered at the origin to fit in the ellipse, its radius cannot exceed the distance of closest point on the ellipse from the origin. So, to locate the appropriate circle, we need to locate the closest point on the ellipse to the origin.

The feasible set is closed (actually in this case it equals its boundary) and is compact, and the function to optimize is a polynomial, so the extreme value theorem applies. Extermizers exist.