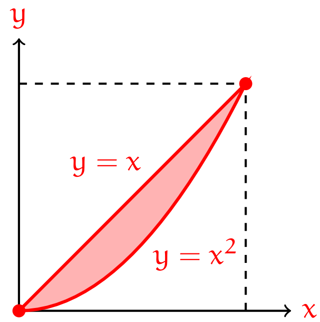
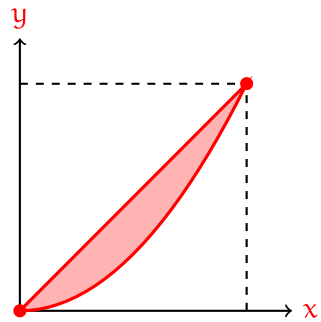
Example 1. Let D be the region bounded by $y = x$ and $y = x^2$ and find:

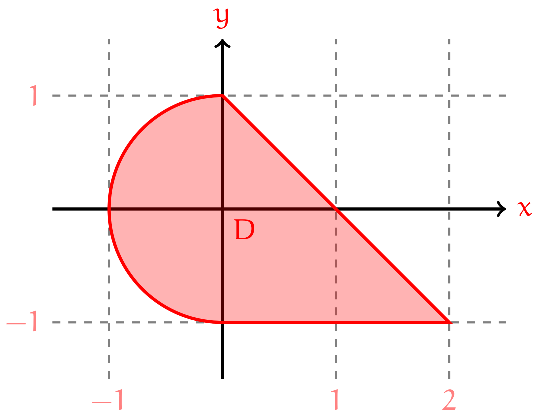




Calculate $\iint_D 2y - x \; dA$ in both orders $dy dx$ and $dx dy$.

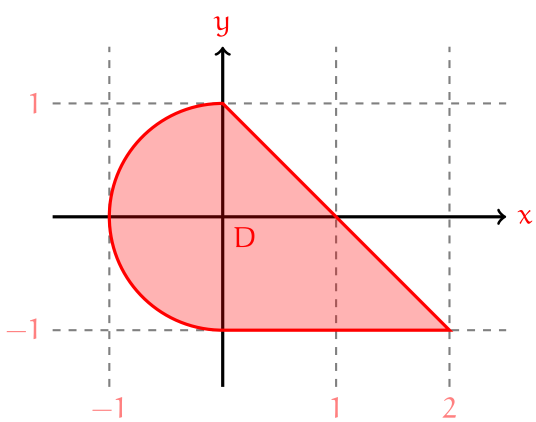
First, let's look at a picture in Desmos.

Example 2. Set up $\iint_D f(x,y)\,dA$ using an iterated integral in both orders.



The left curved edge is the left half of the unit circle.

(a) in order: $dx\,dy$



Order $dy\,dx$ will require a sum of integrals, as we can detect when we observe that the top edge and bottom edge each really consists of two different edges, one comes from half of the circle, and the other part is a straight line.

(b) in order: $dy\,dx$

A. **Area and Density.** We can interpret double integrals in other ways, besides volume under a surface. The idea is to think of $f(x, y)$ as a **weight** assigned to area at (x, y) in the region, so that the double integral is a **weighted sum** of areas.

Do not be afraid to have to think about concepts in multiple ways. With multiple perspectives you will get a greater glimpse at the concealed whole.

The weight could for example indicate the density of a material that makes up that region. For example, in some parts of the region, the density might be twice as much as in others, so in those parts the weight could be 2, while in other parts the weight would be 1.

Double Integral as Mass. If $f(x, y)$ represents the **density** of [mass/electric–charge/...] per unit area at (x, y) in the region D, then:

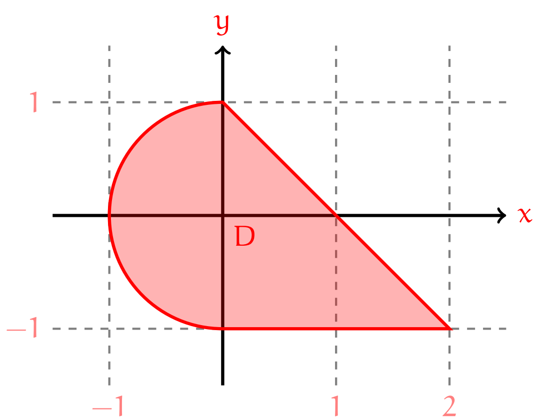
$$\iint_D f(x, y)\, dA =$$

In particular:

$$\iint_D 1\, dA =$$

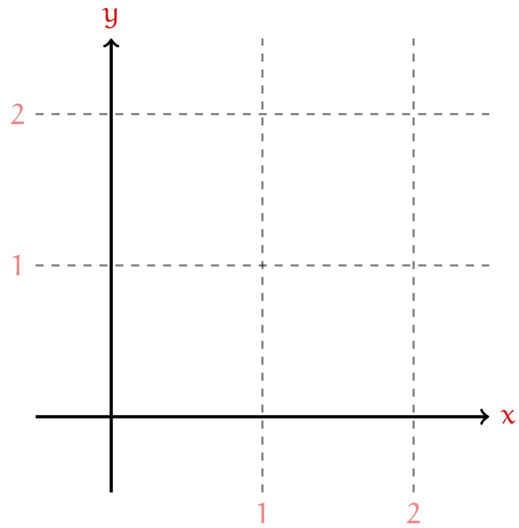
For the last equality, imagine this. If the density of mass per unit area is 1, then the ratio of mass to area is 1, meaning the mass of the region has the same value as the area of the region.

Example 3. Find $\iint_D 1\, dA$ for the region D sketched below:



The left curved edge is the left half of the unit circle.

Example 4. Find $\int_0^2 \int_y^2 e^{x^2}\, dx dy$.



There are certain functions whose antiderivatives are not expressible using familiar functions. A classic example is e^{t^2} which, uh-oh, looks like is the first thing we would need to find an antiderivative for. So we must approach this integral differently. The simplest change of approach is to change the order of integration.

Example 5. Find the integral by changing order:

$$\int_0^3 \int_0^{9-x^2} \frac{xe^{3y}}{9-y}\, dy\, dx$$

Another classic example of a function for that does not have an antiderivative involving familiar functions is e^t/t which, more or less, is like the first function we would need to find an antiderivative for. Hence the suggestion to change order.