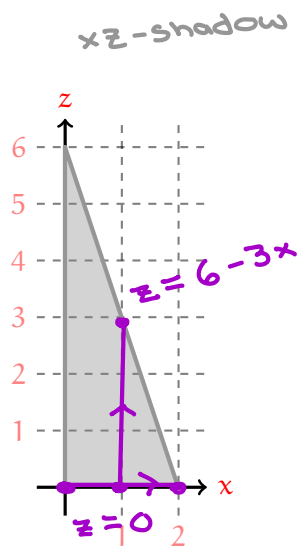
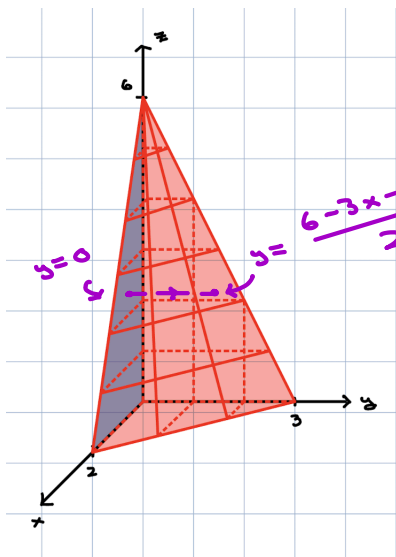


Example 1. Find the volume of the tetrahedron T bounded by the planes: $3x + 2y + z = 6$, $x = 0$, $y = 0$, and $z = 0$, but in the order: $dydzdx$. Last time, we did this in a different order of integration, and found answer 6.



$$\text{volume} = \int \int_T \int 1 \, dV = \int_0^2 \int_0^{6-3x} \int_0^{\frac{6-3x-z}{2}} 1 \, dy \, dz \, dx$$

\leftarrow Fix x, z : y -segment
 τ_{xz} -shadow

$$= \int_0^2 \int_0^{6-3x} \frac{1}{2} (6-3x-z) \, dz \, dx$$

$$= \int_0^2 \left. \frac{1}{2} (6-3x)z - \frac{1}{4} z^2 \right|_{z=0}^{6-3x} dx$$

$$= \int_0^2 \left[\frac{1}{2} (6-3x)^2 - \frac{1}{4} (6-3x)^2 \right] dx$$

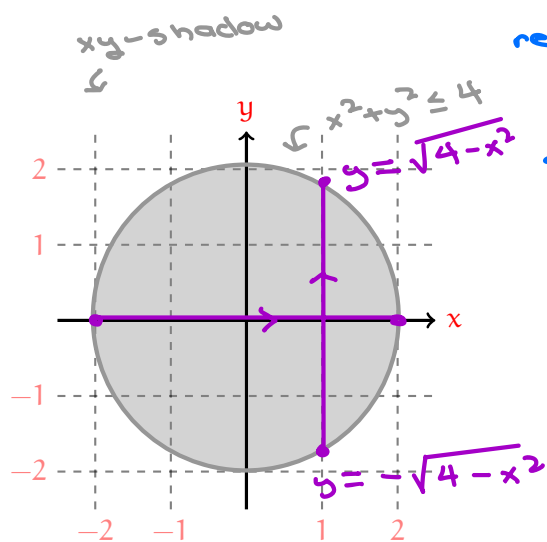
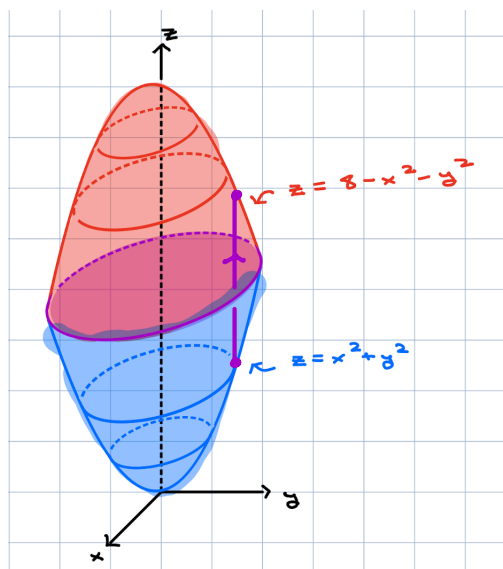
$$= \int_0^2 \frac{1}{4} (6-3x)^2 dx$$

$$\left\{ \begin{array}{l} u = 6 - 3x \rightarrow du = -3 dx \\ x = 0 \rightarrow u = 6 \\ x = 2 \rightarrow u = 0 \end{array} \right.$$

$$= \int_0^6 \frac{1}{12} u^2 du$$

$$= \frac{1}{36} \cdot 6^3 = 6$$

Example 2. If E is the region bounded by the paraboloids $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$ then set up an iterated integral of an arbitrary function $f(x, y, z)$ over E in the order $dzdydx$.



region thickest where the paraboloids intersect:

$$8 - x^2 - y^2 = x^2 + y^2$$

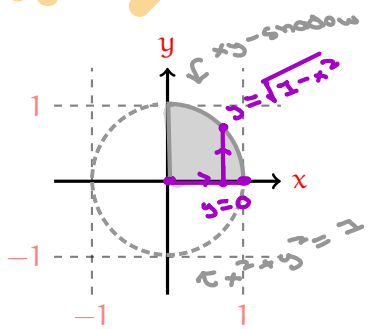
$$8 = 2x^2 + 2y^2$$

$$4 = x^2 + y^2$$

$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{x^2+y^2}^{8-x^2-y^2} f(x, y, z) \, dz \, dy \, dx$$

Example 3. Sketch the region of integration for:

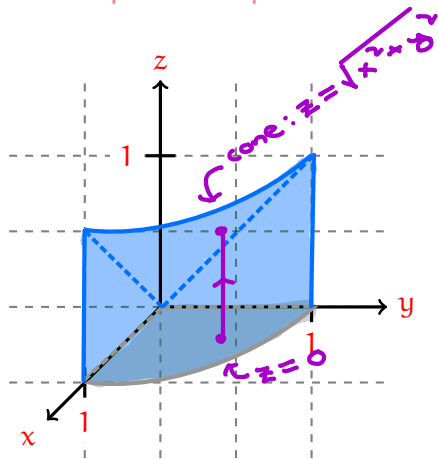
$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{x^2+y^2}} f(x, y, z) \, dz \, dy \, dx$$



↳ xy-shadow:

$$\begin{cases} 0 \leq y \leq \sqrt{1-x^2} \\ 0 \leq x \leq 1 \end{cases}$$

The equation $z = \sqrt{x^2 + y^2}$ describes the top-half of a vertical double cone, i.e. describes an upwards cone, while the equation $y = \sqrt{1-x^2}$ describes the top-half of the unit circle in the xy -plane. We can also view the region in [Desmos](#).



$$\sum 0 \leq z \leq \sqrt{x^2 + y^2}$$

note: $z = \sqrt{x^2 + y^2} \rightarrow z^2 = x^2 + y^2$



E: solid
 → under cone
 → above quarter-disk