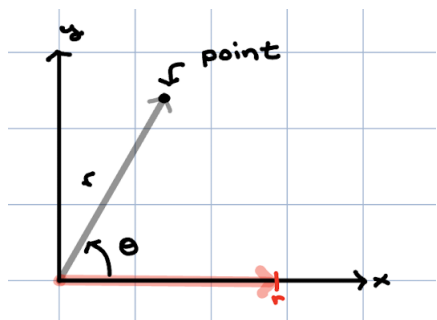


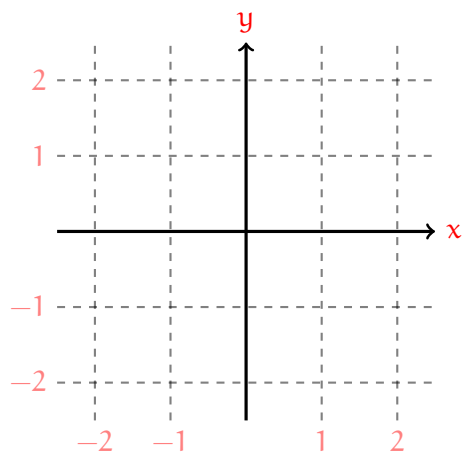
A. **Polar Coordinates.** The **standard** coordinates x and y are not always the most convenient coordinates to describe a particular region in the xy -plane. A useful alternative is captured by **polar coordinates**.



When r is positive, r equals the distance of the point from the origin, and θ measures the counterclockwise angle of the position vector of the point from the positive x -axis.

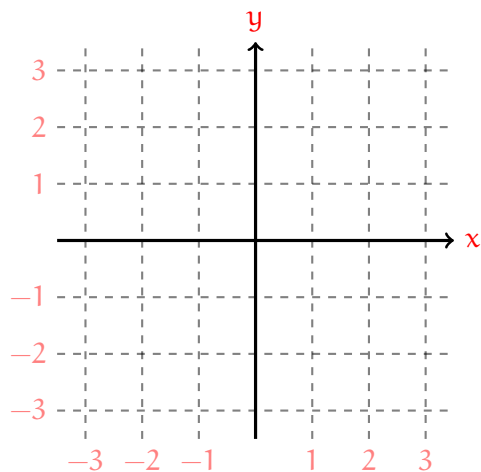
The point with polar coordinates r and θ has position vector obtained by rotating the vector $\langle r, 0 \rangle$ counterclockwise by angle θ .

Sketch the point with polar coordinates $r = -2$ and $\theta = \frac{\pi}{2}$.



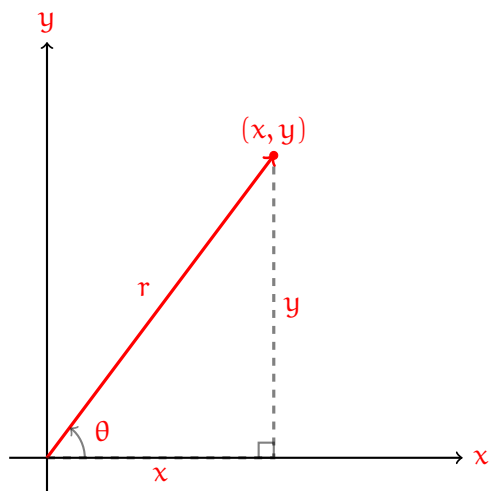
Let's use [Desmos](#) to view the level curves of the polar coordinates.

Example 1. Sketch the region determined by $2 \leq r \leq 3$ and $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.



B. Polar Conversion. We convert between standard and polar coordinates.

By standard coordinates are the x - and y -coordinates.



Key Relations Between Polar and Standard Coordinates.

x in terms of polar:

y in terms of polar:

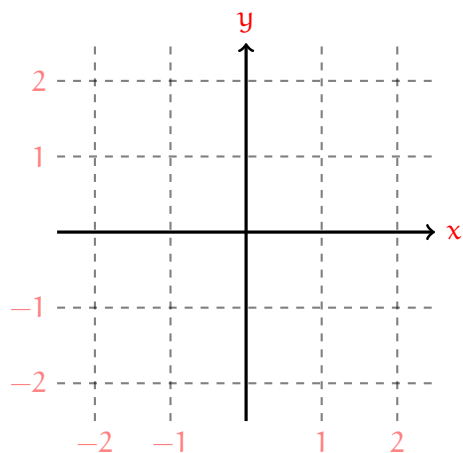
relation between r and standard:

relation between θ and standard:

When r is nonnegative we may write:

$$r = \sqrt{x^2 + y^2}$$

Example 2. Graph $r = 2 \cos \theta$ by first converting to standard coordinates.

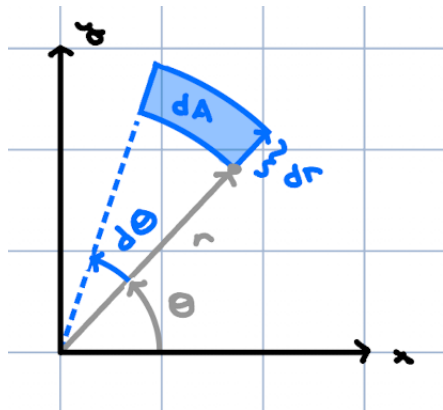


The right side is quite nearly the expression for $2x$ in terms of polar, however the r is missing in front of $\cos \theta$. Our first step is introducing the missing r .

The next step will involve completing the square:

$$x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$

C. Polar Integration. Now we will discuss polar integration. Imagine the infinitesimal bit of area dA obtained from the point with polar coordinates r and θ by increasing θ by $d\theta$ and r by dr .



In polar coordinates:

$$dA =$$

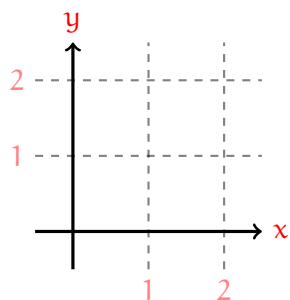
The goal in this process is to try to understand how small changes dr and $d\theta$ are related to small changes dA in area. This dA is exactly what appears in double-integration.

For example, we know that small changes dx and dy effect the change $dA = dx dy$ in area, essentially because the picture we would sketch in this case would be a rectangle with sides dx and dy .

It is a standard fact from geometry that the arc of a circle of radius r traced out by a positive angle θ , measured in radians, has length equal to $r \cdot \theta$.

Example 3. Let D be the region in the first quadrant bounded by $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

The first quadrant is defined by $x \geq 0$ and $y \geq 0$.



and find: $\iint_D e^{x^2+y^2} dA$