Example 1. Let E be the solid cylinder $x^2 + y^2 \leqslant 9$ and $0 \leqslant z \leqslant 4$. Let $\mathcal{S}$ be the boundary of E, oriented with **inward** normals. If:

$$\vec{F} = \left\langle e^{y^2 z^2}\ ,\ y + \sin(e^{x+z})\ ,\ z^2 - z \right\rangle$$

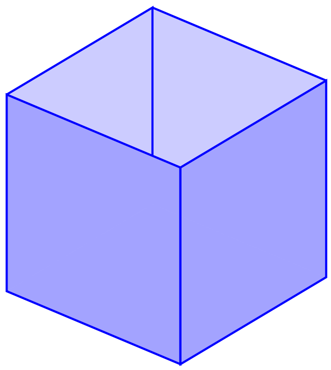
then find $\oiint_{\mathcal{S}} \vec{F} \cdot d\vec{S}$

Remember that if you want to compute a triple integral with cylindrical coordinates, then $dV = r\ dz dr d\theta$.

Example 2. Let $\mathcal{S}$ be the surface of the box:

$$[-1,1] \times [-1,1] \times [0,2]$$

but without its top, and oriented with **outward** normals.



Use the divergence theorem to find the flux of $\vec{F}$ across $\mathcal{S}$ where:

$\vec{F} = (2x + y + z)\vec{\imath} + (x + 2y - z)\vec{\jmath} + (x + y + 2z)\vec{k}$

This surface is not closed, as the paper–cut boundary of the surface is given by the top edges. This is an issue, because the divergence theorem only applies to closed surfaces. Devastating. One strategy in this scenario is to **close off** the surface, by introducing a new surface $\mathcal{S}'$ that, combined with $\mathcal{S}$, forms a closed surface. In our case we will simply include the missing top–face $\mathcal{S}'$. Since $\mathcal{S}$ is oriented with outward normals, we also orient $\mathcal{S}'$ with outward normals. For the top–face, outwards means upwards!

Remember that parameters x and y we have:

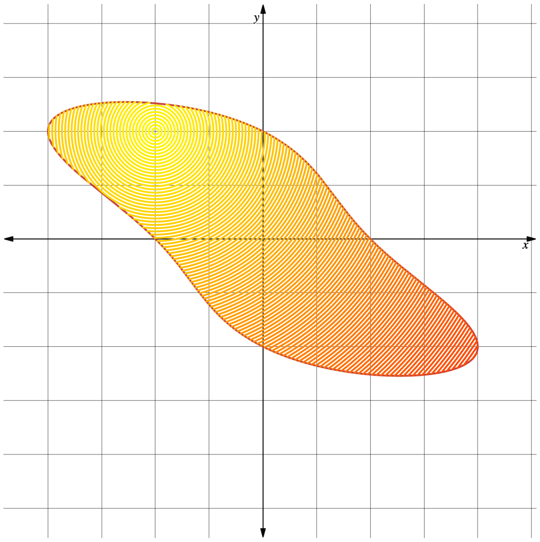
$d\vec{S} = \pm\langle -\frac{\partial z}{\partial x}, -\frac{\partial z}{\partial y}, 1\rangle \, dx dy$

A. **Center of Mass.** Consider a **lamina** situated in the xy–plane and with:

A **lamina** is an infinitesimally thin plate, with mass distributed throughout.

[density of mass at (x, y)] $\to \delta(x, y)$

[total mass] $\to M$



For the purpose of several physical applications, the lamina behaves like a concentrated point mass with mass M, where the location of that point mass is called the **center of mass**.

Another way to describe the center of mass is the point at which we can horizontally balance the lamina on the tip of a pencil.

To locate it, we take the average value of that coordinate over points of the lamina, weighted by the fraction of total mass concentrated at each point. For example:

$\bar{x} =$ [x–coordinate of center of mass]

The **moment** of a coordinate variable is:

[moment of coordinate] $=$

The corresponding coordinate value for the center of mass is: