

**Math 226 Final Exam**  
**December 11th, 2019**

**Directions:** Before starting the exam:

- Write down your name and your student ID.
- Check the box next to the class for which you are registered.
- Read the following rules and then sign.

You must **show all of your work and justify your methods** to obtain full credit. Simplify your final answers and then circle them. **If you use scratch paper, please make sure that all of the work that you want graded is included in the relevant portion of the exam packet.** You have been provided with plenty of blank space in the exam packet for this reason (you have two pages for each of the last two problems).

No calculators are allowed, but you may use the double sided HANDWRITTEN sheet of notes that you brought with you. This may be no more than one sheet of  $8\frac{1}{2} \times 11$  paper. Remember, USC considers cheating to be a serious offense; the minimum penalty is failure for the course. Cheating includes “straying eyes” and failing to stop writing when told to do so at the end of the exam.

**Name (please print):** \_\_\_\_\_

**Signature:** \_\_\_\_\_

**Student ID:** \_\_\_\_\_

<input type="checkbox"/> N. Bottman 9 AM	<input type="checkbox"/> N. Bottman 10 AM	<input type="checkbox"/> R. Sacker 11 AM
<input type="checkbox"/> N. Tiruvilumala 10 AM	<input type="checkbox"/> N. Tiruvilumala 11 PM	<input type="checkbox"/> A. Mazel-Gee 12 PM
<input type="checkbox"/> W. Ozanski 12 PM	<input type="checkbox"/> W. Ozanski 1 PM	

1 (10 pts)	6 (15 pts)
2 (10 pts)	7 (10 pts)
3 (10 pts)	8 (12 pts)
4 (10 pts)	9 (13 pts)
5 (10 pts)	

**100 Points Total**

1. (10 points) Consider the two parallel planes  $x + y + 3z = 7$  and  $x + y + 3z = 9$ .
  - (a) (5 points) Find the distance between these two planes.

8. (12 points) Calculate

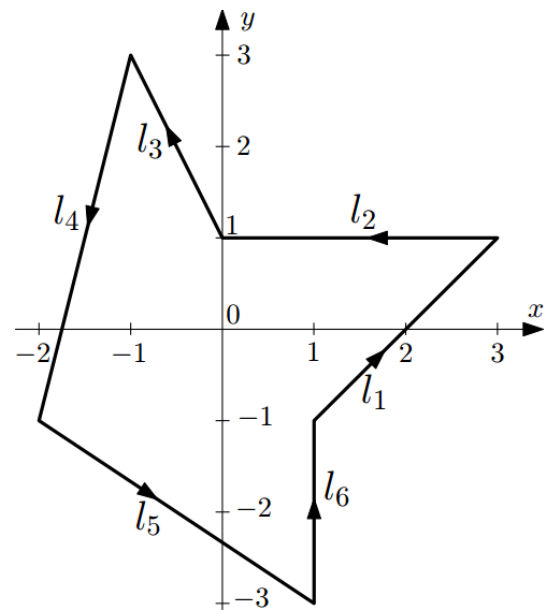
$$\int_{l_2} \mathbf{F} \cdot d\mathbf{r} + \int_{l_3} \mathbf{F} \cdot d\mathbf{r} + \int_{l_4} \mathbf{F} \cdot d\mathbf{r} + \int_{l_5} \mathbf{F} \cdot d\mathbf{r} + \int_{l_6} \mathbf{F} \cdot d\mathbf{r},$$

where

$$\mathbf{F} = \langle -y, x \rangle,$$

and the oriented line segments  $l_1, \dots, l_6$  are as in the figure below. Note that  $\int_{l_1} \mathbf{F} \cdot d\mathbf{r}$  is not included in the above sum.

Hint: You may use without justification the fact that the area enclosed by the figure is 12.



Name: \_\_\_\_\_ Student ID: \_\_\_\_\_

**Directions.** Fill out your name and student ID number on the lines above **right now** before starting the exam! Also, check the box next to the class for which you are registered.

 9am Haskell 11am Tokorcheck 2pm Reardon 10am Haskell 12pm Tokorcheck

- **You must show all your work and justify your methods to obtain full credit.** Do not use scratch paper; if more space is needed, use the extra page provided on the back of the test. If you write on this page, let the grader know that there is work to be found there by writing the page number where it says “MY SOLUTION CONTINUES ON PAGE \_\_\_\_\_”
- Do not write outside the margins.
- Simplify your answers to a reasonable degree. Any fraction should be written in lowest terms. Known trig identities should be simplified. You need not evaluate expressions such as  $\ln 5$ ,  $e^{0.7}$  or  $\sqrt{226}$ .
- No calculators are allowed. **Turn off your cell phone.**
- You may use the sheet of notes that you brought with you, this may be no more than one sheet of  $8\frac{1}{2} \times 11$ ” paper. You may have anything written on it (on both sides), but it must be written in your own handwriting.
- Remember, USC considers cheating to be a serious offense; the minimum penalty is failure for the course. Cheating includes “straying eyes” and failing to stop writing when told to do so at the end of the exam.

1 (14 pts)	2 (12 pts)
3 (10 pts)	4 (10 pts)
5 (10 pts)	6 (10 pts)
7 (10 pts)	8 (12 pts)
9 (20 pts)	10 (12 pts)

**120 points total**

**Question 2.** Let  $f(x, y) = 2y + \cos \pi x - \sqrt{x}$ . Consider the point  $P = (4, -1)$ .

(a) In which direction does  $f$  increase the fastest starting from  $P$ ?

(b) Find the rate of change of  $f$  at  $P$  in the direction of  $\frac{4}{5}\hat{\mathbf{i}} + \frac{3}{5}\hat{\mathbf{j}}$ .

(c) Find an equation of the tangent plane of  $f$  at  $P$ .

(d) Use linear approximation to estimate the value of  $f$  at  $Q = (4.4, -1.1)$ .

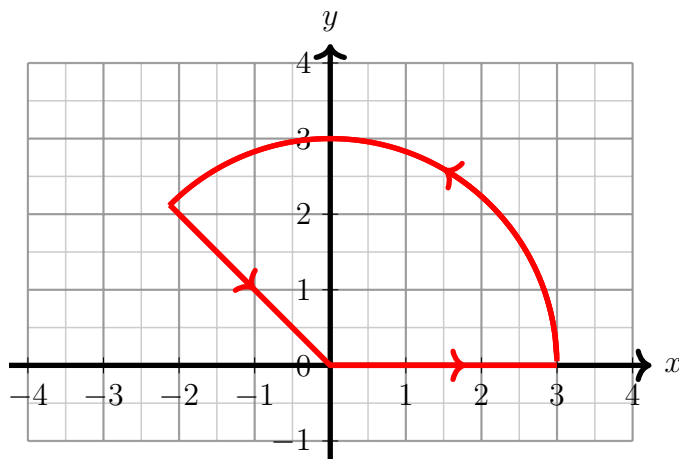
(e) Suppose also that  $x = -4s \cos t$  and  $y = se^{2t}$ . Find  $\frac{\partial f}{\partial s}$  at  $(s, t) = (-1, 0)$ .

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**Question 6.** Compute

$$\int_C (y^2 + xe^x) dx + (x^2y + 2xy + \sin(y^2)) dy$$

where  $C$  is the counterclockwise oriented piecewise curve comprised of  $y = \sqrt{9 - x^2}$ ,  $y = -x$ , and  $y = 0$  as shown in the graph below.



MY SOLUTION CONTINUES ON PAGE \_\_\_\_\_

**Question 8.** Consider the vector field

$$\mathbf{F}(x, y, z) = \langle e^z, x + z, e^{xy+z} \rangle$$

and the goldfish bowl surface  $S$  given by

$$x^2 + y^2 + (z + 1)^2 = 2 \quad \text{and} \quad z \leq 0$$

oriented so that the normal points from inside the bowl to outside the bowl. Calculate

$$\iint_S \text{Curl } \mathbf{F} \cdot d\mathbf{S}.$$

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