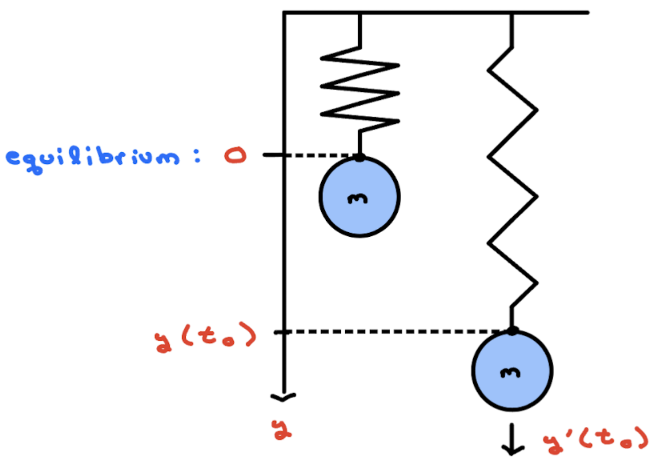
A. **Differential Equations.** The displacement $y(t)$ down from equillibrium of a mass hanging on a spring at time t satisfies differential equation:

$$my''(t) + \mu y'(t) + ky(t) = 0$$

A **differential equation** is an equation involving derivatives. The differential equation modeling the spring comes from physical experimental observations.

While the details are not important now: k is the **spring constant** and measures the spring's stiffness, and μ is **damping constant** and measures the resistance to motion of the medium (e.g. air)



If at time t_0 we are provided **initial values**:

initial position:

initial velocity:

A differential equation with initial values is referred to as an **initial value problem**.

Then theoretically we can find the **solution**:

Our differential equation is **ordinary** because it involves:

ODE is the shorthand for an ordinary differential equation.

Because the maximal derivative of the dependent variable y that occurs is:

our differential equation is said to be:

Example 1. Solve the initial value problem:

$y' = t^2 e^{t^3 - 1}$ with $y(1) = 0$

Here y' is shorthand for $y'(t)$ or $\frac{dy}{dt}$.

This is a **first–order** ODE because it only involves the first derivative.

The expression in terms of arbitrary constant C is called the **general solution**.

Example 2. Find the general solution to the **separable** differential equation:

$$y' = \frac{xy + x}{x - 1}$$

Here it is implied that $y' = \frac{dy}{dx}$.

A differential equation is **separable** if variables can be separated:

$y' = f(x)g(y)$

which is a very special thing indeed.

The general strategy is:

1. Write in form $\frac{1}{g(y)}\,dy = f(x)\,dx$.
2. Integrate: $\int \frac{1}{g(y)}\,dy = \int f(x)\,dx$.
3. Solve for y.

Do not forget: include an integration constant C in Step 2.