A. **Slope Fields.** Let us return to the differential equation:

$$y' = \frac{xy + x}{x - 1}$$

and sketch its **slope field**.

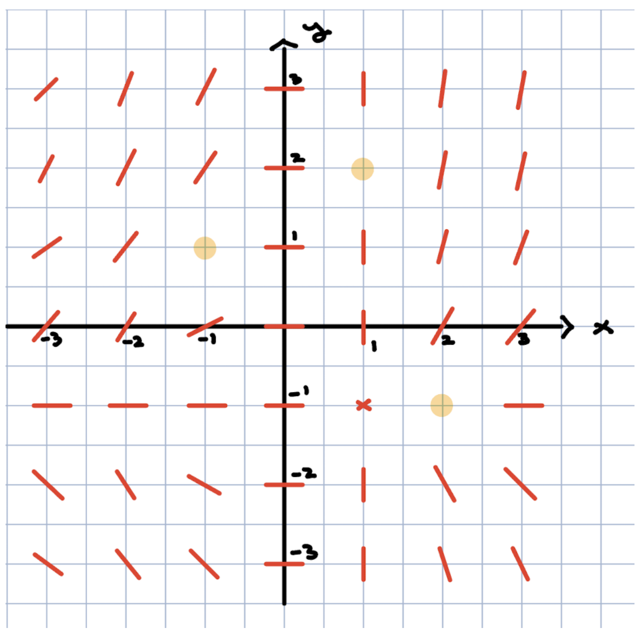
Think of y' as the slope of the solution y—the derivative is a slope after all!

Our differential equation has the form:

$y' = [\text{function of } x \text{ and } y]$

so each point (x, y) gives a slope y' which we can sketch as a sloped line segment.

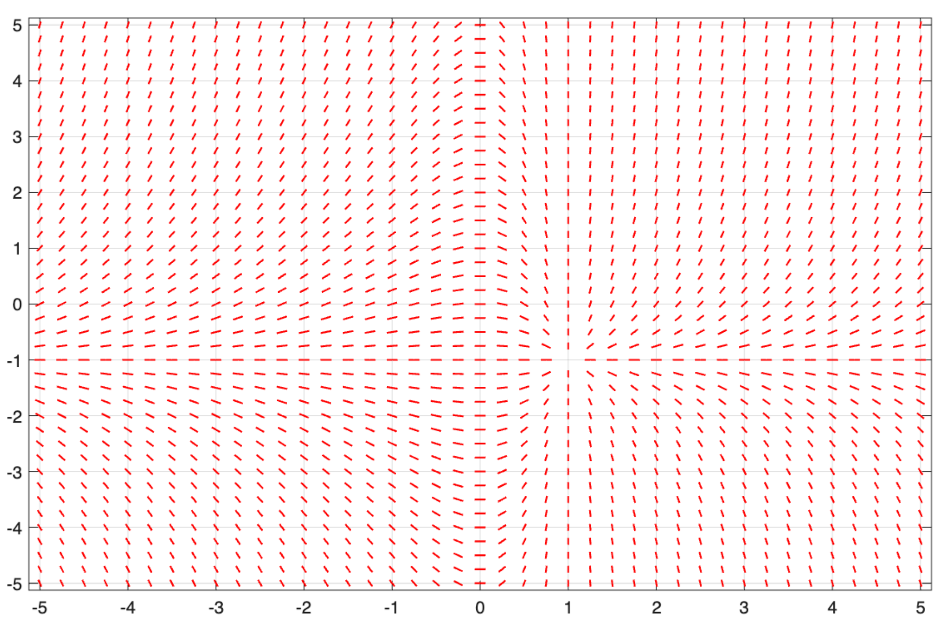
Sometimes the slope approaches $\pm\infty$, in which case we sketch a vertical segment. On difficult days, sometimes the slope of my life approaches ∞, in a bad way.



Earlier we had solved the differential equation, finding:

$$y = Ae^x(x - 1) - 1$$

Use the more detailed slope field below to sketch the solution with initial value: $y(0) = 1$



I made this slopefield using MATLAB. One of your future assignments will be to execute exactly this kind of task. It is, like, so pretty, omigosh.

Example 1. Solve the initial value problem and state the interval of existence.

$y' = \dfrac{e^x}{1+y}$ with $y(0) = -2$

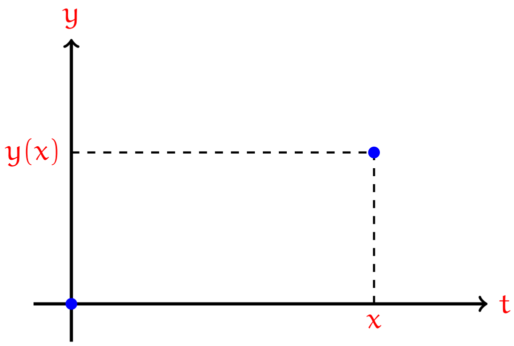
The **interval of existence** is the maximum interval containing the initial input on which the solution is defined. I hope my interval of existence is large.

Do you remember the quadratic formula for $ax^2 + bx + c$?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

It never stays away for long.

Example 2. Find functions $y(x)$ with the property that: the area under the curve $y = y(t)$ from 0 to x is always equal to one–fourth the area of the rectangle with vertices at $(0, 0)$ and $(x, y(x))$.



The key to this word problem is to set it up as an initial value problem, and then solve that problem. The key to life is to first accept things as they are, and from that foundation, effect positive change.