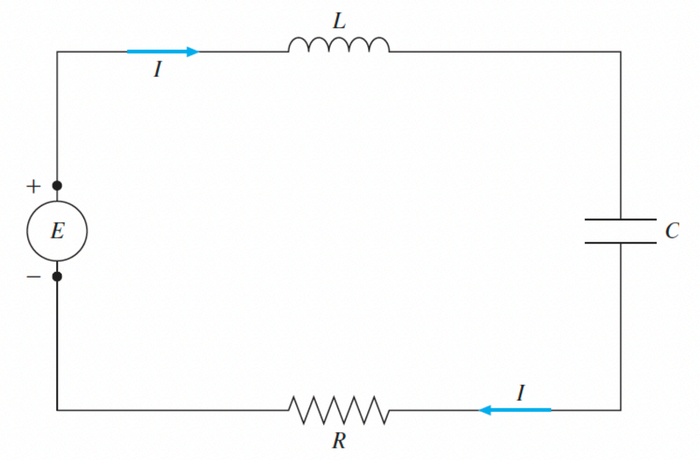
A. **Electrical Circuits.** Consider an ideal electrical circuit consisting of a simple loop including a voltage source E, a resistor R, a capacitor C, and an inductor L. Denote the electrical current by I.



A **voltage source** could be a battery. Its effectiveness is measured by its **voltage** E which has units **volts**.

A **resistor** impedes the current. Its effectiveness is measured by its **resistance** R is a constant that has units **ohms** (Ω).

A **capacitor** stores electrical energy by accumulating charge on two closely spaced surfaces separated by an insulator. Once enough charge builds up, the current reverses the direction of its flow. There is no current passing through an ideal capacitor. Its effectiveness is measured by its **capacitance** C which has units **farads** (F)

A typical **inductor** is a coil of wire. Current flowing through creates a magnetic field that opposes changes in the current. Its effectiveness is measured by its **inductance** L which is measured in **henrys** (H).

The **voltage** across a component is the work required to move a positive charge across that component. Voltage causes electrons, hence electric charge Q (in **coulombs**) to flow through the circuit. The rate of flow is the **current**:

$I =$

and is measured in **amperes** (A).

Laws Governing Voltage of Components.

Ohm's Law. The voltage E_R across a resistor satisfies:

$E_R =$

Faraday's Law. The voltage E_L across an inductor satisfies

$E_L =$

Capacitance Law. The voltage E_C across a capacitor satisfies:

$E_C =$

Remember that Q denotes charge: in this case the charge built up on the capacitor.

Kirkhoff's Voltage Law.

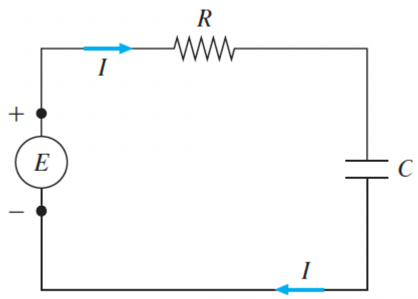
The directed sum of voltages around any closed loop equals 0. Therefore:

$E =$

The word **directed** here means that we fix a direction around the loop to calculate voltage, the work required to move positive charge across components.

Example 1. An RC circuit has voltage source with voltage $E = 2\cos t$ V, a resistor with resistance $R = 2\ \Omega$, and an capacitor with capacitance $C = \frac{1}{6}$ F.

An RC circuit only contains a voltage source, resistor, and capacitor.



Find a formula for current $I(t)$ assuming the the initial current is 0.

Remember that current is the derivative of charge: $I = \frac{dQ}{dt}$. Also: Ohm's Law $E_R = RI$ and the Capacticance Law $E_C = \frac{Q}{C}$.

Integration by parts can be used to show:

$$\int e^{at}\cos bt\ dt = \frac{e^{at}\left(a\cos bt + b\sin bt\right)}{a^2+b^2}$$

$$\int e^{at}\sin bt\ dt = \frac{e^{at}\left(-b\cos bt + a\sin bt\right)}{a^2+b^2}$$

B. **Bernoulli Equations.**

A **Bernoulli equation** is a differential equation that can be put into form:

$y' + a(x)y =$

Bernoulli equations are **nonlinear** unless $n = 0$ or $n = 1$. So linear methods cannot be used directly.

We can convert a Bernoulli equation to a linear equation as follows:

If we use substitution:

$u =$

then the Bernoulli equation at the top of the page can be transformed to:

Example 2. Solve the differential equation:

$xy' + y = xy^2$

Remember that if we select $u = y^{1-n}$ then the Bernoulli equation:

$y' + a(x)y = f(x)y^n$

is transformed to:

$u' + (1-n)a(x)u = (1-n)f(x)$