A. **Undetermined Coefficients.** We now suppose we have **nonzero** forcing term:

Remember: the forcing term is the righthand side. We are still working with 2nd–order linear differential equations with constant coefficients. Just no longer homogeneous.

$y'' + py' + qy = f(t)$

Just like in the first–order linear setting the general solution looks like:

$y = [\text{particular solution}] + [\text{general homogeneous solution}]$

$y =$

Here y_{h1} and y_{h2} are fundamental homogeneous solutions to the associated $y'' + py' + qy = 0$.

The new difficulty is identifying the particular solution.

Undetermined Coefficients: Polynomial–Exponential.

If the forcing term has form:

$f(t) = \Big[\text{degree } m \text{ polynomial}\Big] \cdot e^{at}$

For example: $f(t) = (t^2 + t + 1)e^{3t}$.

Then attempt undetermined **trial solution** of form:

A trial solution is a solution we will try out. The trial solution will have... undetermined coefficients. Which we need to determine.

$y_p =$

where $n =$

Example 1. Find the general solution to:

$y'' - 3y' - 4y = 36te^{2t}$

Example 2. Find a particular solution to the differential equation:

$y'' - 3y' - 4y = 2e^{-t}$

Based on previous discussion: the appropriate trial solution for forcing term $f(t) = [\text{constant}]e^{kt}$ would be $t^n e^{at}$ where n is the number of times that a is a root of the characteristic equation.

B. **Complex Undetermined Coefficients.** We now suppose we have forcing term:

$\Big[\text{degree } m \text{ polynomial}\Big] \cdot e^{at}\cos bt$

or: $\Big[\text{degree } m \text{ polynomial}\Big] \cdot e^{at}\sin bt$

We use that:

$e^{at}\cos bt =$

$e^{at}\sin bt =$

Remember that:

$e^{(a+ib)t} = e^{at}e^{ibt} = e^{at}\cos bt + ie^{at}\sin bt$

Undetermined Coefficients: Trig.

If we have forcing term equal to a **linear combination** of:

$$= \Big[\text{degree } m \text{ polynomial}\Big] \cdot e^{at}\cos bt$$

$$= \Big[\text{same degree } m \text{ polynomial}\Big] \cdot e^{at}\sin bt$$

Either: use **complex method** by using **complex** forcing term:

$F(t) =$

and then selecting **complex trial solution** of form:

$z_p =$

where $n =$

and then as appropriate obtaining final answer:

$y_p =$

Or: use **real method** by selecting trial solution of the form:

$y_p =$

Specifically: if the forcing term is the linear combination $f = A\mathrm{Re}(F) + B\mathrm{Im}(F)$ then we select $y_p = A\mathrm{Re}(z_p) + B\mathrm{Im}(z_p)$.

Example 3. Find the general solution to the differential equation:

$y'' - y = 2t\cos t$

Complex Method.

Example 3 continued. . . Revisiting the same differential equation:

$y'' - y = t\cos t$

instead set up the trial solution y_p for using the **real method.**