A. **Linearity.**

Linearity of Laplace Transforms. If a and b are constant:

$$\mathcal{L}\left\{af(t) + bg(t)\right\}(s) =$$

This is tied to the linearity of integrals.

$$\int_0^\infty e^{-st}\left(af(t) + bg(t)\right)\, dt =$$

$$a\left(\int_0^\infty e^{-st} f(t)\, dt\right) + b\left(\int_0^\infty e^{-st} g(t)\, dt\right)$$

Example 1. Find formulas for:

$$\mathcal{L}\{\cos bt\}(s) =$$

$$\mathcal{L}\{\sin bt\}(s) =$$

$$\mathcal{L}\{\cos bt\}(s) =$$

$$\mathcal{L}\{\sin bt\}(s) =$$

Remember that:

$e^{ibt} = \cos bt + i \sin bt$

Because the Laplace transform is linear (and transforms real–valued functions to real–valued functions) it can be shown that it preserves real parts and imaginary parts. That is:

$\mathcal{L}(\operatorname{Re} f) = \operatorname{Re} \mathcal{L}(f)$ and $\mathcal{L}(\operatorname{Im} f) = \operatorname{Im} \mathcal{L}(f)$

B. **Shift Theorem.** Find:

$\mathcal{L}\left\{e^{at}f(t)\right\}(s) =$

Shift Theorem. If a is constant then:

$\mathcal{L}\left\{e^{at}f(t)\right\}(s) =$

This says that multiplication by e^{at} in the t–domain corresponds to a shift by a in the s–domain.

Example 2. Find formulas for:

$\mathcal{L}\{e^{at}\cos bt\}(s) =$

$\mathcal{L}\{e^{at}\sin bt\}(s) =$

$\mathcal{L}\{t^n e^{at}\}(s) =$

Recall that:

$\mathcal{L}\{\cos bt\}(s) = \dfrac{s}{s^2+b^2}$

$\mathcal{L}\{\sin bt\}(s) = \dfrac{b}{s^2+b^2}$

$\mathcal{L}\{t^n\}(s) = \dfrac{n!}{s^{n+1}}$

C. **Time Multiplication.** Find the relationship between:

$\mathcal{L}\{f(t)\}(s) =$

$\mathcal{L}\{tf(t)\}(s) =$

Time Multiplication. If n is a positive integer then:

$\mathcal{L}(t^n f) =$

The idea is multiplying by t in the t–domain leads to multiplication by $-\frac{d}{ds}$ in the s–domain. So multiplication n times by t in the t–domain leads to mult. by $(-1)^n \frac{d^n}{ds^n}$ in the s–domain.

Example 3. Find:

$\mathcal{L}\left\{e^t(t+1)\cos 3t\right\}(s) =$

Remember: $\mathcal{L}\{e^{at}\cos bt\} = \dfrac{s-a}{(s-a)^2+b^2}$