A. **Derivative in Time Domain.** Find:

$\mathcal{L}\{y'(t)\}(s) =$

The fact that:

$\lim_{t\to\infty} e^{-st}f(t) = 0$

is part of the requirement that the improper integral be defined, since $e^{-st}f(t)$ is the integrand. If this were not so then the improper integral would yield infinite area.

$\mathcal{L}(y'')(s) =$

Laplace Transform of Derivative.

Let Y be the Laplace transform of $y(t)$. Then:

$\mathcal{L}\left(y^{(n)}\right)(s) =$

Example 1. Find the Laplace transform Y of the solution to the IVP:

$y'' - 3y' - 10y = 2$ with $y(0) = 1$ and $y'(0) = 2$

B. **Inverse Laplace Transforms.**

Definition. If $\mathcal{L}(f) = F$ then the **inverse Laplace** of F is:

That this can be defined is tied to the fact that the Laplace transform is one–to–one, meaning distinct function have distinct Laplace transforms.

Linearity. Like the Laplace transform, its inverse is linear:

$$\mathcal{L}^{-1}\left(aF + bG\right) =$$

Here a and b represent constants.

To find the inverse Laplace it will help to have on hand the following table.

Table of Laplace Transforms.

$f(t)$	$\mathcal{L}\{f(t)\}(s)$
1	$\dfrac{1}{s}$
t^n	$\dfrac{n!}{s^{n+1}}$
e^{at}	$\dfrac{1}{s-a}$
$t^n e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}$
$\cos bt$	$\dfrac{s}{s^2+b^2}$
$\sin bt$	$\dfrac{b}{s^2+b^2}$
$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$
$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$

Example 2. Find formulas for:

Assuming n is a positive integer and a and b are constant:

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-a)^n}\right\}(t) =$$

$$\mathcal{L}^{-1}\left\{\frac{s-a}{(s-a)^2+b^2}\right\}(t) =$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-a)^2+b^2}\right\}(t) =$$

Recall:

$$\mathcal{L}\{t^n e^{at}\}(s) = \frac{n!}{(s-a)^{n+1}}$$

$$\mathcal{L}\{e^{at}\cos bt\}(s) = \frac{s-a}{(s-a)^2+b^2}$$

$$\mathcal{L}\{e^{at}\sin bt\}(s) = \frac{b}{(s-a)^2+b^2}$$

Example 3. Calculate:

$$\mathcal{L}^{-1}\left\{\frac{2}{(s+3)^5} + \frac{2s-3}{s^2-2s+5}\right\}(t) =$$

You should have done a MyOpenMath review on completing the square:

$x^2 + bx = (x + \frac{b}{2})^2 - \frac{b^2}{4}$

We only apply it here because $s^2 - 2s + 5$ is **irreducible**, meaning it cannot be factored using real numbers. This can be detected by using the quadratic formula.