A. **Partial Fraction Decomposition.** To calculate inverse Laplace transforms of general rational functions we need to execute partial fraction decompositions.

Partial Fraction Decomposition Review.

A proper rational function:

$$\frac{p(s)}{q(s)}$$

can be decomposed as a sum according to the **factors** of its **denominator**.

factor $(s-a)^n$ contributes summand:

irreducible factor $(s-a)^2+b^2$ contributes summand:

A **rational function** is a fraction of polynomials.

Partial fraction decomposition converts a rational function into a some of simpler rational functions.

A rational function is **proper** if the degree (largest present power) of the numerator is smaller than the degree of the denominator.

Irreducible means it cannot be factored further using real numbers. For example s^2+4. You can check irreducibility using the quadratic formula: does it only have imaginary roots?

Example 1. Calculate the inverse Laplace of:

$$F(s) = \frac{4}{s^4 - 4s^3 + 4s^2}$$

There are two main approaches. The 1st is comparing coefficients. The 2nd is plugging in roots (and, if repeated roots, taking derivatives then plugging in roots).

Recall the formula:

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-a)^n}\right\}(t) = \frac{t^{n-1}e^{at}}{(n-1)!}$$

Example 2. Calculate the inverse Laplace of:

$$F(s) = \frac{10s}{(s^2-1)(s^2+2s+2)}$$

Recall the formulas:

$$\mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\}(t) = e^{at}$$

$$\mathcal{L}^{-1}\left\{\frac{s-a}{(s-a)^2+b^2}\right\}(t) = e^{at}\cos(bt)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-a)^2+b^2}\right\}(t) = \frac{e^{at}\sin(bt)}{b}$$