Example 1. Use Laplace transforms to solve the following initial value problem.

$y'' - 3y' - 10y = 2$ with $y(0) = 1$ and $y'(0) = 2$.

Recall the rule:

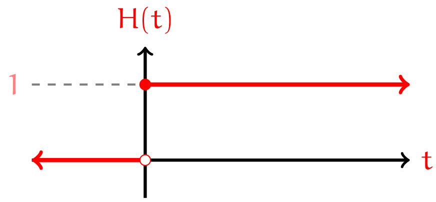
$\mathcal{L}(y^{(n)}) = s^n Y - s^{n-1}y(0) - \cdots - y^{(n-1)}(0)$

$\mathcal{L}(1) = \dfrac{1}{s}$

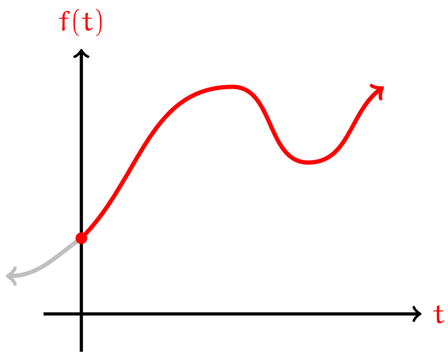
A. **Heaviside Function.**

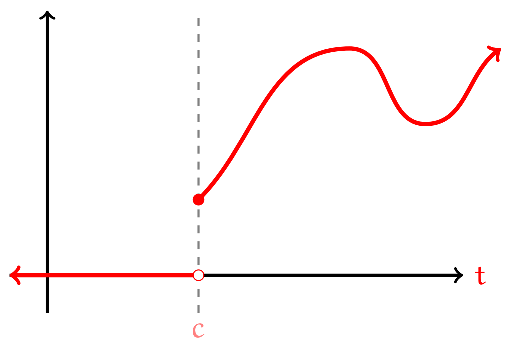
The **Heaviside function** is:

$$H(t) = \begin{cases} 0 \text{ if } t < 0 \\ 1 \text{ if } t \geqslant 0 \end{cases}$$



The shift–right–by–c of the part of $f(t)$ with $t \geqslant 0$ is sketched below:





Find: $\mathcal{L}\left\{H(t-c)f(t-c)\right\}(s) =$

Translation Formula.

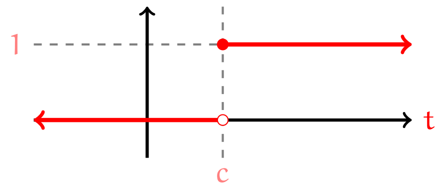
$\mathcal{L}\left\{H(t-c)f(t-c)\right\}(s) =$

B. **Truncation.**

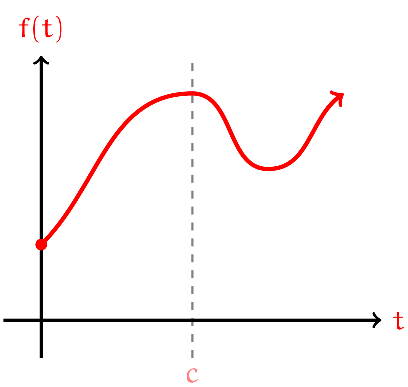
The **truncated Heaviside** for $t \geqslant c$ is sketched below:

$$H_c(t) = \begin{cases} 0 \text{ if } t < c \\ 1 \text{ if } t \geqslant c \end{cases}$$

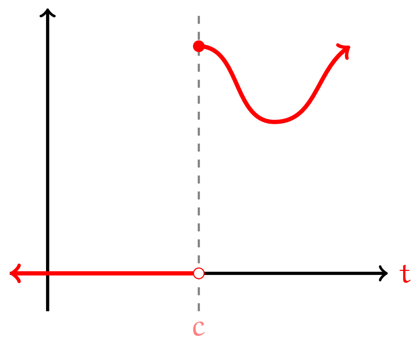
$H_c(t) =$



The **truncation** of $f(t)$ for $t \geqslant c$ is sketched below:



$f_c(t) =$



Truncation Formula.

$$\mathcal{L}\left\{f_c(t)\right\} = \mathcal{L}\left\{H_c(t)f(t)\right\} = \mathcal{L}\left\{H(t-c)f(t)\right\} =$$

The idea is that we have shown:

$\mathcal{L}\{H(t-c)f(t-c)\} = e^{-cs}\mathcal{L}\{f(t)\}$

That is: multiplying by $H(t-c)$ in the t–domain is the same as adding c in the t–domain and multiplying by e^{cs} in the s–domain.

Example 2. Find:

$\mathcal{L}\left\{H\left(t-\frac{\pi}{4}\right)\sin(t)\right\}(s) =$

Recall that:

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

Example 3. Find a formula for:

$$\boxed{\mathcal{L}\left\{H_c(t)\right\}(s) =}$$

$\mathcal{L}\left\{H_c(t)\right\}(s) =$