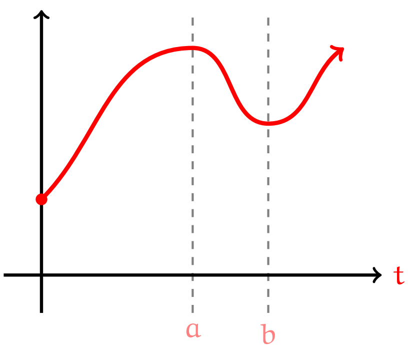
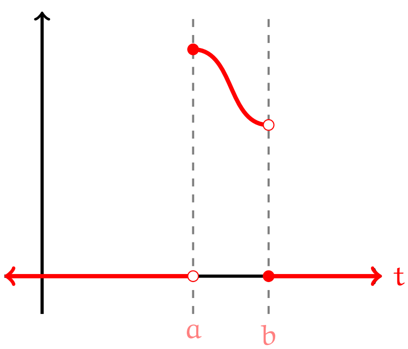
A. **Pulses.** A pulse is a temporary arising of a function.

The **pulse** of $f(t)$ for $a \leqslant t < b$ is sketched below:



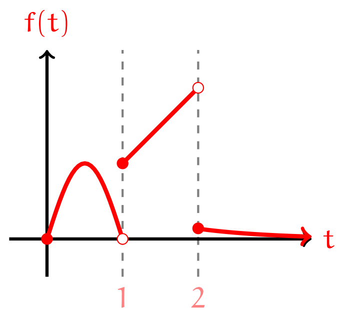
$f_{ab}(t) =$



$f_{ab}(t) =$

Example 1. Without computing any integrals, find the Laplace transform of:

$$f(t) = \begin{cases} \sin \pi t & \text{if } 0 \leqslant t < 1 \\ t & \text{if } 1 \leqslant t < 2 \\ e^{-t} & \text{if } t \geqslant 2 \end{cases}$$



The strategy for piecewise functions:

Step 1. Write $f(t)$ terms of Heavisides by thinking of it as a sum of pulses.

Step 2. Use the truncation formula:

$$\mathcal{L}\left\{H_c(t)f(t)\right\} = e^{-cs}\mathcal{L}\{f(t+c)\}$$

Step 3. Rewrite the translated functions as needed using rules like:

$$e^{A+B} = e^A e^B$$

$$\sin(A+B) = \sin A \cos B + \sin B \cos A$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

Step 4. Compute Laplace transforms using your table of Laplace transforms. In this example we use:

$$\mathcal{L}\{\sin bt\} = \frac{b}{s^2 + b^2}$$

$$\mathcal{L}\{t^n\} = \frac{n!}{t^{n+1}}$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

B. **Inverse Translation.**

Inverse Translation Formula.

$$\mathcal{L}^{-1}\left\{e^{-cs}F(s)\right\} =$$

Recall the translation formula said:

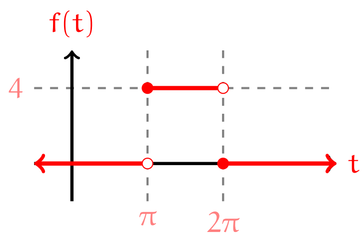
$\mathcal{L}\{H(t-c)f(t-c)\} = e^{-cs}F(s)$

Example 2. Use Laplace transforms to solve:

$y'' + 4y = f(t)$ with $y(0) = 1$ and $y'(0) = 0$

where:

$$f(t) = \begin{cases} 0 & \text{if } 0 \leqslant t < \pi \\ 4 & \text{if } \pi \leqslant t < 2\pi \\ 0 & \text{if } t \geqslant 2\pi \end{cases}$$



and express your answer as a piecewise function.

Recall we had found:

$\mathcal{L}\{H_c(t)\}(s) = \dfrac{e^{-cs}}{s}$

Recall the formulas:

$$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+b^2}\right\} = \cos bt$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+b^2}\right\} = \frac{\sin bt}{b}$$