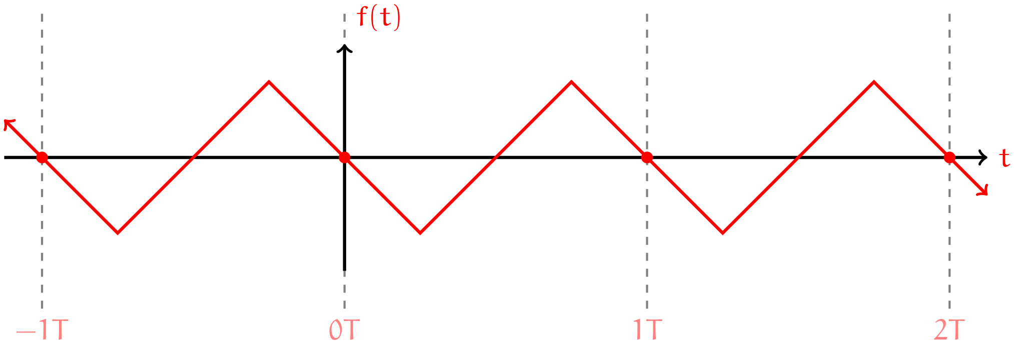
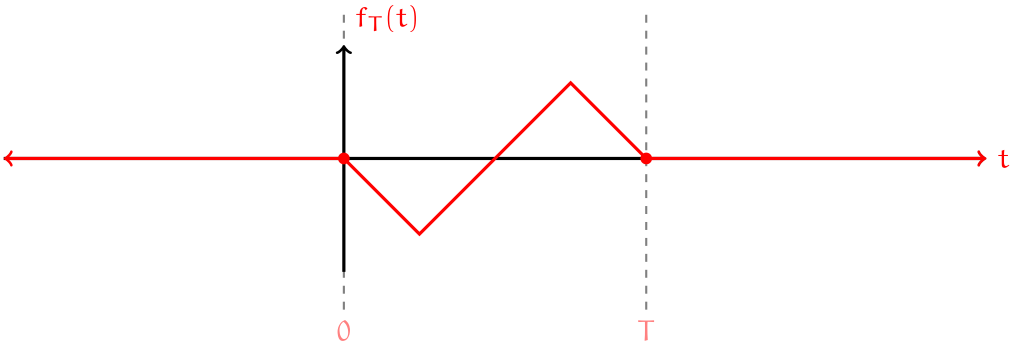
A. **Periodic.** Consider a **periodic** function $f(t)$ with period T.

To have period T means T is the smallest number so that $f(t+T) = f(t)$ for all t.



Its **window** is the pulse: $f_T(t) =$



Here is an optional derivation of the formula we will use:

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt = \sum_{n=0}^{\infty} \left[\int_{nT}^{nT+T} e^{-st} f(t)\, dt \right] = \cdots$$

We divide the integral into periods.

$$\cdots = \sum_{n=0}^{\infty} \left[\int_{nT}^{nT+T} e^{-st} f_T(t-nT)\, dt \right] = \cdots$$

Here we think of the segment of $f(t)$ from nT to $nT+T$ as the shift by nT of the window $f_T(t)$.

$$\cdots = \sum_{n=0}^{\infty} \left[\int_0^T e^{-s(u+nT)} f_T(u)\, du \right] = \sum_{n=0}^{\infty} \left[e^{-snT} \int_0^T e^{-su} f_T(u)\, du \right] = \cdots$$

Here we execute u–substitution $u = t - nT$ and simplify.

$$\cdots = \sum_{n=0}^{\infty} \left[e^{-sT} F_T(s) \right] = F_T(s) \sum_{n=0}^{\infty} e^{-snT} = F_T(s) \sum_{n=0}^{\infty} (e^{-sT})^n = \cdots$$

$$\cdots = \frac{F_T(s)}{1 - e^{-sT}}$$

Here we note:

$$\int_0^T e^{-su} f_T(u)\, du = \mathcal{L}(f_T)(s)$$

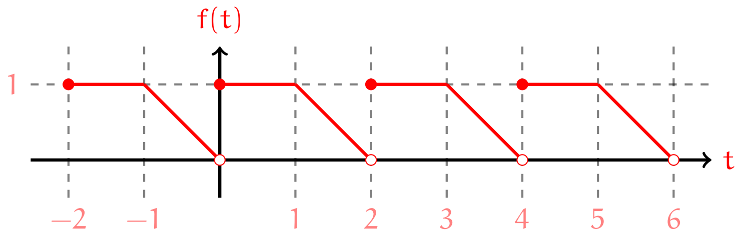
and we call this $F_T(s)$. Then we simplify using the geometric sum formula:

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

Periodic Laplace. If $f(t)$ is periodic with window $f_T(t)$ then:

$\mathcal{L}\{f(t)\}(s) =$

Example 1. Find the Laplace transform of the periodic function $f(t)$ graphed below.



Recall that the pulse of a function $f(t)$ from $a \leqslant t < b$ equals:

$[H_a(t) - H_b(t)] \cdot f(t)$

Recall the formulas:

$\mathcal{L}\{H_c(t)\}(s) = \dfrac{e^{-cs}}{s}$

$\mathcal{L}\{H_c(t)f(t)\} = e^{-cs}\mathcal{L}\{f(t+c)\}$

$\mathcal{L}\{t\}(s) = \dfrac{1}{s^2}$

Recall the periodic formula:

$\mathcal{L}\{f(t)\} = \dfrac{F_T(s)}{1 - e^{-sT}}$