A. **Convolutions.** Suppose:

$$\mathcal{L}\{f(t)\}(s) = F(s) = \int_0^\infty e^{-st} f(t)\ dt$$

$$\mathcal{L}\{g(t)\}(s) = G(s) = \int_0^\infty e^{-st} g(t)\ dt$$

In the context above: the **convolution** of f and g is the function $f * g$ so:

$\mathcal{L}(f * g) =$

$\mathcal{L}^{-1}(F \cdot G) =$

In other words, the convolution in the t–domain corresponds to multiplication in the s–domain. Note that $*$ here is **not** normal multiplication!

We find a formula for the convolution by rewriting:

$F(s)G(s) =$

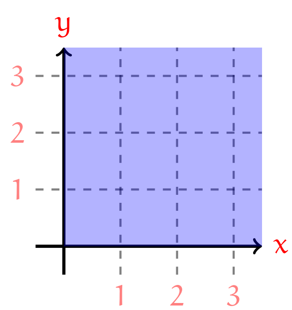
We use x and y in place of using the same t for the two integrals to avoid confusion.

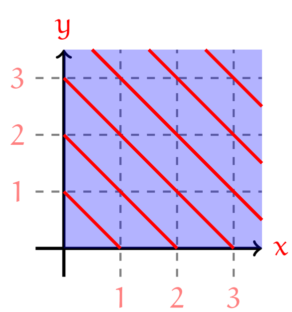
The idea is that we then treat this as a double–integral over the shaded region. Remember double integrals? We then execute a change of variables:

$t = x + y$

$u = x$

and then write the integral in order $dudt$. Due to technical principles of multivariable change of variables, it will be the case that $dudt = dxdy$. If the outer variable $t = x + y$ is held constant, then the inner variable $u = x$ varies from 0 to t.





Convolution Formula.

$(f * g)(t) =$

The idea is that we have written:

$F(s)G(s) = \int e^{-st}[\text{something}]\ dt$

and so $F(s)G(s)$ must be the Lagrange transform of that [something], which means that something is the convolution.

Example 1. Let $f(t) = \cos t$ and $g(t) = \sin t$ and find:

$(f * g)(t) =$

In order to complete this you will need the product–to–sum identity:

$\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$

Do not worry: on an exam you would be provided such an identity if needed.

Example 2. Use convolutions, not partial fractions, to compute the inverse Laplace:

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2-5s+6}\right\} =$$

The idea is to write this as a product in the s–domain, which we can convert to a convolution in the t–domain.