A. **Eigenbasis.**

An **eigenbasis** for a 2×2 matrix A consists of:

Not all 2×2 matrices have eigenbases.

We will be very interested in finding eigenbases for the purposes of solving systems of differential equations.

Example 1. Attempt (and fail) to find an eigenbasis for:

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}$$

Remember, an eigenbasis requires two linearly independent eigenvectors, meaning two eigenvectors that are not scalar multiples of each other.

B. **Complex Eigenstuff.** A real matrix A can have a complex eigenvalue:

$\lambda = a + bi$ with complex eigenvector $\mathbf{v} = \mathbf{a} + i\mathbf{b}$

Apply the complex conjugate to:

$A\mathbf{v} = \lambda\mathbf{v}$

Because polynomials can sometimes have complex roots.

Complex eigenvalues and eigenvectors come in complex conjugate pairs:

$\lambda = a + bi$ with eigenvector $\mathbf{v} = \mathbf{a} + i\mathbf{b}$

$\overline{\lambda} =$ with eigenvector $\overline{\mathbf{v}} =$

Example 2. Find a complex eigenbasis for:

$$A = \begin{pmatrix} -1 & 2 \\ -2 & -1 \end{pmatrix}$$