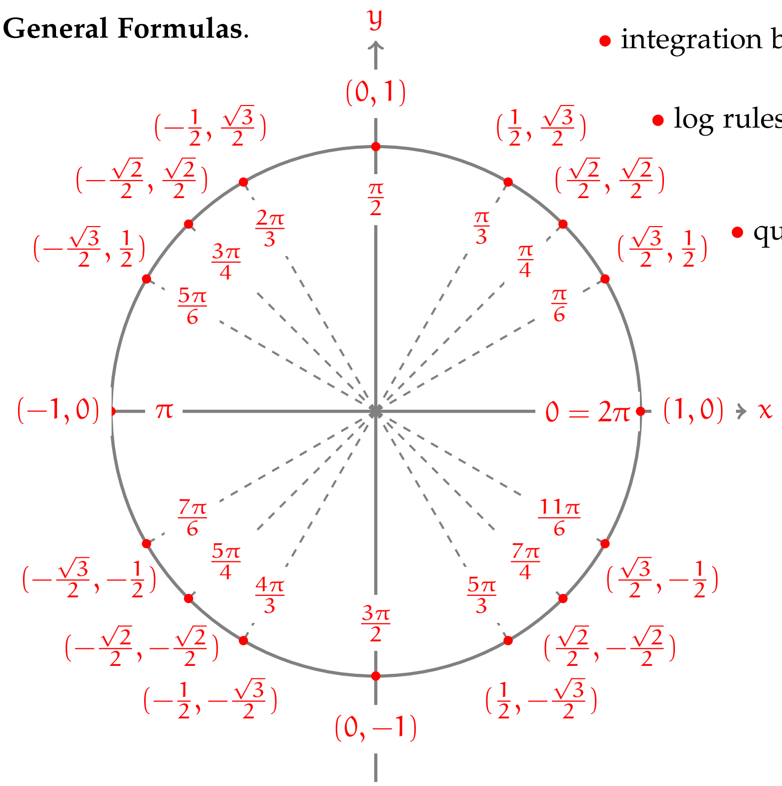
Math 245 – Formula Sheet.

General Formulas.



- integration by parts: $\int u \, dv = uv - \int v \, du$
- log rules: $\ln(A) + \ln(B) = \ln(AB)$, $\ln(A) - \ln(B) = \ln\left(\frac{A}{B}\right)$,
  $c\ln(A) = \ln\left(A^c\right)$
- quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

A2 Formulas.

- for $y' + a(x)y = f(x)$ we have:
  - integrating factor $I(x) = e^{\int a(x)\,dx}$
  - variation of parameters: $y = uy_h$
    - where $y_h = e^{-\int a(x)\,dx}$ and $u = \int \frac{f(x)}{y_h}\,dx$
- circuits
  - Kirkhoff's Voltage Law: $E = RI + LI' + Q/C$
  - derivative of charge is current: $I = Q'$
- Bernoulli equation: $y' + a(t)y = f(t)y^n$
  - substitute $u = y^{1-n}$
  - converts to $u' + (1-n)a(t)u = (1-n)f(t)$

A3 Formulas.

- spring with no external force: $my'' + \mu y' + ky = 0$
- amplitude-phase: $a\cos(\omega t) + b\sin(\omega t) = A\cos(\omega t - \phi)$
  - $A = \sqrt{a^2 + b^2}$ and $(A\cos\phi, A\sin\phi) = (a, b)$
- Wronskian: $W(y_1, y_2) = y_1 y_2' - y_1' y_2$
- Abel's Theorem: for $y'' + p(t)y' + q(t)y = 0$
  - $W(y_1, y_2) = Ae^{-\int p(t)dt}$ if y_1, y_2 are solutions
  - if y_1 is a solution ,hen Abel's theorem for $A = 1$ yields:
    - so is $y_2 = uy_1$ where $u = \int \frac{e^{-\int p(t)dt}}{y_1^2}\,dt$