Example 1. The pdf of a continuous random variable X is given below.

$$f(x) = \begin{cases} C \cdot (4x - 2x^2) & \text{if } 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the value of the constant C.

(b) Find the cumulative distribution function $F(x)$ of X.

Example 2. Let X be a continuous random variable and let $Y = aX + b$, where $a > 0$ and b are constants. Find the cdf and pdf of Y in terms of the cdf and pdf of X.

In discussion, you will do the same, but with $a < 0$.

A. **Independence.** For **discrete** random variables X and Y, we said they were independent if, for all x and y, we had:

$\mathbb{P}(X = x, Y = y) = \mathbb{P}(X = x)\mathbb{P}(Y = y)$

For continuous random variables, this is not very useful, because probabilities of single values are 0. Instead, we make an analagous definition using cumulative distribution functions.

Independence of Continuous Random Variables. Two random variables X and Y are **independent** if and only if, for all x and y, we have:

$\mathbb{P}(X \leqslant x, Y \leqslant y) =$

Example 3. Let X be uniform "selection" from the interval $(0, 10)$, followed by an **independent** uniform "selection" Y from the interval $(5, 10)$. Let Z be the maximum of X and Y. What is the probability $Z \leqslant 7$?

B. **Expectation.** Let X be a random variable. In the **discrete** case, the expectation was the expression:

$$\mathbb{E}[X] = \sum_x x \cdot \mathbb{P}(X = x)$$

Evidently, this is useless if we insist that X is a **continuous** random variable, as probabilities of singular values are 0. Instead, we'll use probability over infinitesimal intervals instead, in which case our sum will turn into an integral:

$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x \cdot \mathbb{P}(x \leqslant X \leqslant x + dx) =$$

Here we recall that multiplying the density of probability at x by the length dx of an infinitesimal interval equals the probability, at least for the purposes of computing integrals.

Expectation of Continuous Random Variables. If X is a continuous random variable with pdf $f(x)$, then the **expected value** of X is:

$$\mathbb{E}[X] =$$

Several properties of expectation for discrete random variables carry over for continuous random variables.

Linearity: $\mathbb{E}[aX + bY] = a\mathbb{E}[X] + b\mathbb{E}[Y]$

Independent Product: if X and Y are independent, then $\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y]$

C. **Expectation: Continuous Uniform.** Let X be uniform on (a, b). We had found its probability density function to be:

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a < x < b \\ 0 & \text{otherwise} \end{cases}$$

Then its expected value is:

$$\mathbb{E}[X] =$$

Expectation: Continuous Uniform. If X is uniform on (a, b), then:

$$\mathbb{E}[X] =$$