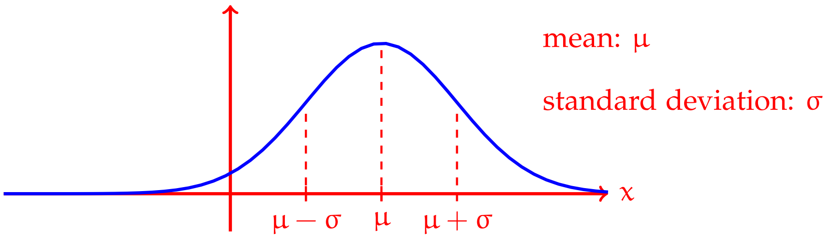
A. **General Normal Distribution.** We can adjust the normal distribution to have a different mean μ and standard deviation σ. The formula for the density function is spoiled in the graph, but let's derive it.

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



We will **show** that the transformation $\mathcal{N} = \sigma X + \mu$ of the **standard** normal random variable X has the bell curve pdf given above, and has mean μ and standard deviation σ.

Using the formula for the pdf of $aX + b$ derived in an earlier example (whic applies for any random variable X, as long as a is positive), the pdf of $\mathcal{N}$ is:

That formula said the pdf of $aX + b$, if $a > 0$, was:

$$\frac{1}{a} \cdot f\left(\frac{x-b}{a}\right)$$

and the cdf is:

$$F\left(\frac{x-b}{a}\right)$$

$f_{\mathcal{N}}(x) =$

and its cdf is:

Recall the pdf of the standard normal is:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

and the cdf is $\Phi(x)$.

$F_{\mathcal{N}}(x) =$

And further:

$\mathbb{E}[\mathcal{N}] =$

$\mathrm{Var}(\mathcal{N}) =$

Recall that the second moment of the standard normal random variable is $\mathbb{E}[X^2] = 1$.

$\mathrm{SD}(\mathcal{N}) =$

General Normal Random Variable. The normal random variable with mean μ and standard deviation σ is:

$$\mathcal{N}(\mu, \sigma^2) = \mu X + \sigma \text{ where } X \text{ is the standard normal random variable}$$

Note the parameters in $\mathcal{N}(\mu, \sigma^2)$ are its mean μ and its **variance** σ^2.

It has pdf:

$$f_{\mathcal{N}}(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

and cdf:

$\mathbb{P}(\mathcal{N} \leqslant x) =$

Example 1. Let $\mathcal{N}$ be a normal random variable with mean $\mu = 5$ and variance $\sigma^2 = 100$. Find:

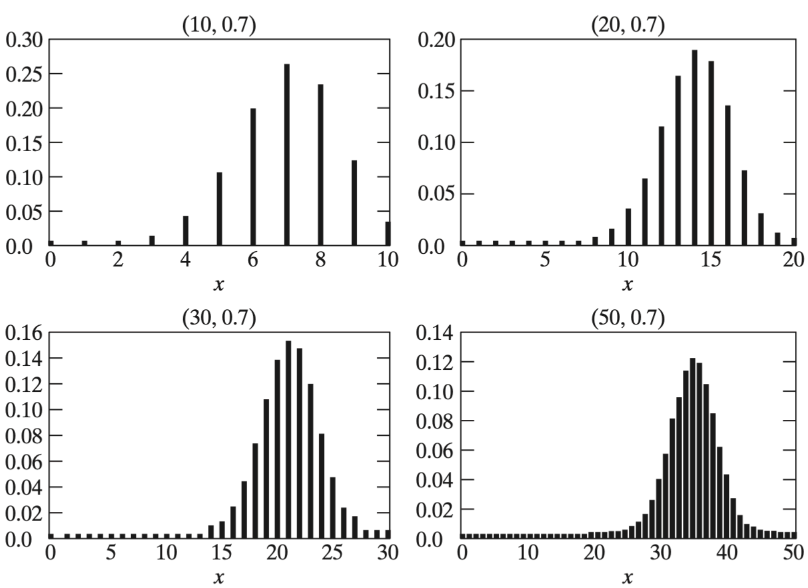
$\mathbb{P}(-6 \leqslant \mathcal{N} \leqslant 7) =$

B. **Normal Approximation of Binomial.** The bell curve shape naturally appears as a limit involving many independent trials of an experiment. Here is one such example. Let X be a binomial random variable, counting the number of successes in n independent trials of an experiment with probability of success p. Recall:

$\mu = \mathbb{E}[X] =$

$\sigma = \sqrt{\mathrm{Var}(X)} =$

Then, when n is large, by a theorem called the **Central Limit Theorem** that we will see much later in the class, the cdf of X can be approximated by the cdf of $\mathcal{N}(\mu, \sigma)$.



Normal Approximation of Binomial. Let X be a binomial random variable with mean μ and standard deviation σ. Then, if n is large, the cdf of X can be approximated by the cdf of $\mathcal{N}(\mu, \sigma)$. Thus:

$\mathbb{P}(X \leqslant x) \approx \mathbb{P}(\mathcal{N} \leqslant x) =$

Example 2. The ideal class size in a college is 150. From past experience, the college knows about $1/3$ of admitted students accept. For this reason, the college admits 450 students. Let X be the resulting class size. Approximate the following by using a normal approximation with **continuity correction**, which takes advantage of the fact that X takes on **integer values** to replace restrictions with equal probability restrictions involving $\text{integers} + 0.5$.

$\mathbb{P}(X = 150) =$

$\mathbb{P}(X > 151) =$

$\mathbb{P}(X \leqslant 151) =$