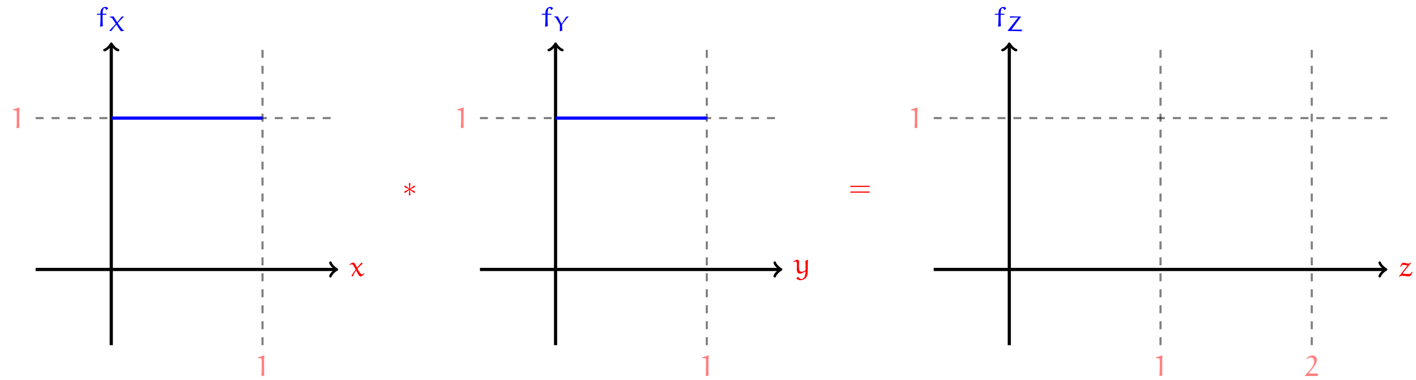
Example 1. Let X and Y be independent uniform random variables on $(0, 1)$. Find the pdf of their sum $Z = X + Y$.



Recall, the convolution of two functions $f(x)$ and $g(y)$ is:

$$(f * g)(z) = \int_{-\infty}^{\infty} f(z - y)g(y)\ dy$$

Example 2. Let $X \sim \text{Exp}(1)$ and $Y \sim \text{Gamma}(\alpha, 1)$ be **independent** random variables with common rate parameter $\lambda = 1$, and the gamma random random variable has shape parameter $\alpha > 0$.

Show that $X + Y \sim \text{Gamma}(\alpha + 1, 1)$.

Recall that the pdf of $\text{Exp}(1)$ is:

$$e^{-t} \text{ for } t > 0$$

and the pdf of $\text{Gamma}(\alpha, 1)$ is:

$$\frac{e^{-t}t^{\alpha-1}}{\Gamma(\alpha)} \text{ for } t > 0$$

Recalla the functional equation for the gamma function:

$\Gamma(\alpha) = (\alpha - 1)\Gamma(\alpha - 1)$ if $\alpha > 1$

Sum of Gammas. This is a special case of a general fact. If $X \sim \text{Gamma}(\alpha, \lambda)$ and $Y \sim \text{Gamma}(\beta, \lambda)$ are **independent** gamma random variables with the same **rate** parameter λ, then their sum:

$X + Y \sim$

A. **Conditional Probability Mass.** Recall that, if A is a **positive** probability event, then the conditioned probability $\mathbb{P}(\cdot \mid A)$ itself defines a probability space. Given the event A, a random variable X becomes a random variable $X \mid A$ that we call X **conditioned on** A, and thus which has its own probability distribution.

In other words, it satisfies all the axioms of a probability space.

Of special interest is when there is another **discrete** random variable Y, and A is the event $Y = y$.

Conditional Probability Mass. Let X and Y be **discrete** random variables.

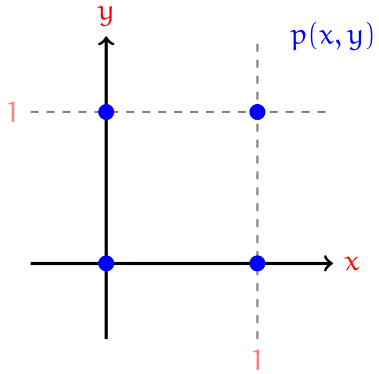
Then the **conditional probability mass function** of X given $Y = y$ is:

$p(x \mid y) =$

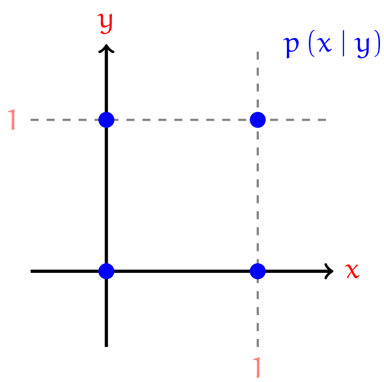
Remember, conditional probability is undefined (but treated as finite, so multiplying by 0 still yields 0) if the event $Y = y$ has probability 0.

Example 3. Let $p(x, y)$ be a joint mass function whose values on its **support** are $p(0, 0) = 0.4$, $p(0, 1) = 0.2$, $p(1, 0) = 0.1$, $p(1, 1) = 0.3$.

Recall the support is where the random variable has nonzero probability.



Find the conditional probability mass function, $p\left(x \mid y\right) =$



Find the conditional probability mass function, $p\left(y \mid x\right) =$