Example 1. Let $X \sim \text{Binom}(n, p)$ and use the tower property to find:

$\mathbb{E}[X \mid X > 0]$

The approach we use here is to condition X on $X > 0$ and its complement $X = 0$.

Example 2. The number of people that come into a store is $N \sim \text{Poisson}(50)$. Each person, independently of N and of all others, spends $X_i \sim \text{Uniform}(0, 20)$ dollars. Let X be the revenue of the store.

Find $\mathbb{E}[X \mid N]$.

Note that $\mathbb{E}[X \mid N]$ is a function of N. We are just trying to find **which** function it is.

Use the tower property to quickly find $\mathbb{E}[X]$.

Recall that:

$\mathbb{E}\left[\text{Poisson}(\lambda)\right] = \lambda$

A. **Conditional Variance.** Let X and Y be random variables, so that Y is discrete or X and Y are jointly continuous.

The **conditional variance** of X given Y is the variance of X, given we have observed Y.

$\mathrm{Var}\,(X \mid Y) =$

Now, the tower property (aka law of total expectation) tells us how to use conditional expectation $\mathbb{E}[X \mid Y]$ to compute unconditioned expectation $\mathbb{E}[X]$. There is also a way to use conditional variance $\mathrm{Var}\,(X \mid Y)$ to compute unconditioned variance $\mathrm{Var}\,(X)$.

Law of Total Variance. $\mathrm{Var}(X) =$

Let us prove the law of total variance, by computing each term, to the point that it is clear the equality holds:

$\mathbb{E}[\mathrm{Var}(X \mid Y)] =$

$\mathrm{Var}(\mathbb{E}[X \mid Y]] =$

Example 3. Let's return to the an earlier example: The number of people that come into a store is $N \sim \text{Poisson}(50)$. Each person, independently of N and of all others, spends $X_i \sim \text{Uniform}(0, 20)$ dollars. Let X be the revenue of the store.

Find $\text{Var}(X \mid N)$.

Note that $\text{Var}(X \mid N)$ is a function of N. We are just trying to find **which** function it is.

Recall that variance is additive over **independent** sums.

Recall that:

$$\text{Var}\left(\text{Uniform}(a, b)\right) = \frac{(b-a)^2}{12}$$

Earlier we found $\mathbb{E}[X \mid N] = 10N$. Use the law of total variance to find $\text{Var}(X)$.

Recall that: $\text{Var}\left(\text{Poisson}(\lambda)\right) = \lambda$