Example 1. Let Z be standard normal. Find its moment generating function.

Recall, the pdf of the standard normal is:

$$f(z) = \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}}$$

A. **Moment Generating Function Properties.** Let's find the moment generating function of a linear expression $aX + b$ of a random variable.

$M_{aX+b}(t) =$

Next let's suppose $X \perp Y$, i.e. that X and Y are **independent**. Let's find the moment generating function of their sum.

$M_{X+Y}(t) =$

Example 2. Let $\mathcal{N}$ be a normal random variable with mean μ and standard deviation σ. Find the moment generating function of X.

Example 3. Let $X \sim \text{Binomial}(n, p)$ be a binomial random variable with parameter p. Find the moment generating function of X.

Example 4. Let $X \sim \text{Gamma}(\alpha, \lambda)$ be a Gamma random variable with parameters α and λ. If α is a positive integer, find the moment generating function of X. Note: The end expression you obtain actually works if α is any real number.

Recall: the moment generating function of an exponential random variable with parameter λ is:

$M_{\text{Exp}(\lambda)}(t) = \dfrac{\lambda}{\lambda - t}$ for $t < \lambda$.

B. **Markov's Inequality.** Our next goal is to establish several foundational probability limit theorems. To do so, we need a toolkit of inequalities that apply for arbitrary random variables.

Suppose X is a $\geqslant 0$ random variable and a is a constant. Let:

$\mathbb{1}_{X \geqslant a} =$ [indicator function of $X \geqslant a$]

Then:

$X \geqslant$

Markov's Inequality. Let X be a $\geqslant 0$ random variable and a be a > 0 constant. Then:

$\mathbb{P}(X \geqslant a) \leqslant$

C. **Chebyshev's Inequality.** We use Markov's inequality to establish another inequality. Suppose X is a random variable with $\mathbb{E}[X] = \mu$ and $\mathrm{Var}(X) = \sigma^2$. Let a be a > 0 constant. Then:

$\mathbb{P}(|X - \mu| \geqslant a) =$

Chebyshev's Inequality. Let X be random variable and a be a > 0 constant. Then:

$\mathbb{P}(|X - \mu| \geqslant a) \leqslant$

Example 5. Let X be a Poisson random variable with $\lambda = 15$. Use Chebyshev's inequality to find a lower bound for:

$\mathbb{P}(11 \leqslant X \leqslant 19)$