A. **Approximating Using the Central Limit Theorem.** Let $X_1, X_2, \ldots$ be independent and identically distributed, with identical distribution to that of random variable X, with mean μ and standard deviation σ.

Then, if n is sufficiently large, dependent on where we would like to consider the distribution, the central limit theorem tells us:

distribution of $\left[\sqrt{n}\cdot\frac{\overline{X}_n-\mu}{\sigma}\right] \approx$ distribution of $\left[\qquad\qquad\right]$

The distribution of a **sum/empirical–average** of many **iid** random variables can be approximated by the distribution of:

Example 1. Let X be a Poisson random variable with $\lambda = 100$. Use the central limit theorem, with continuity correction, to approximate:

$\mathbb{P}(X \geqslant 120)$

Example 2. Let $X_1, \ldots, X_{12}$ be independent and uniform on the interval $(0, 1)$. Let $\overline{X}_{12}$ be the average of these random variables. Use the central limit theorem to approximate:

$\mathbb{P}(\overline{X}_{12} \leqslant 0.6)$

B. **Chernoff Bounds.** There is another useful set of bounds on probability we can obtain, this time in terms of the moment–generating function. We have:

$\mathbb{P}(X \geqslant a) =$

Chernoff Bounds. Let X be a random variable. Then:

$\mathbb{P}(X \geqslant a) \leqslant$ for any $t > 0$

$\mathbb{P}(X \leqslant a) \leqslant$ for any $t < 0$

First, let's show Chernoff bounds are always **concave up** functions of t.

Importantly, this ensures critical points of the Chernoff bounds are:

$\frac{d^2}{dt^2}\left[e^{-ta}\mathbb{E}\left[e^{tX}\right]\right] =$

Example 3. Let X be a Gamma random variable with rate parameter $\lambda = 1$ and shape parameter $\alpha = 2$. Find the **optimal** Chernoff bound for:

$\mathbb{P}(X \geqslant 5)$