Example 1. The arrival process of **returning** and **new** customers to a shop by t hours after opening are independent Poisson process.

Returning customers arrive at a mean rate of 2 per hour, and new customers arrive at a mean rate of 1 per hour.

Let T be the time (in hours) of the first arrival of a customer (returning or new).

$\mathbb{E}[T \mid \text{no customers have arrived in the first half hour}]$

A. **Principle of Thinning.** Suppose that the arrival of vehicular crashes on a highway is a Poisson process $\{Z_t\}_{t \geqslant 0}$ with rate λ.

A given vehicular crash, independent of all else:

$\longrightarrow$ involves a motorcycle with probability p

$\longrightarrow$ does not involve a motorcycle with probability $q = 1 - p$

Let:

$\longrightarrow$ $\{X_t\}_{t \geqslant 0}$ be the counting process of crashes:

$\longrightarrow$ $\{Y_t\}_{t \geqslant 0}$ be the counting process of crashes:

Then, given positive time t, the joint mass function of X_t and Y_t is:

$p(m, n) = \mathbb{P}(X_t = m, Y_t = n) =$

Principle of Thinning. If, in a Poisson process with rate parameter λ, arrivals are, independently of all else, labelled "A" with probability p and labelled "B" with probability $q = 1 - p$, then the arrival processes of A's and B's are **independent** Poisson process with rate parameters:

$\lambda_A =$

$\lambda_B =$

As a consequence:

$\lambda =$

$p =$

$q =$

Note: This idea can be extended if we allow more independent labels, say labels "A_i" with probability p_i, in which case the parameter is $\lambda_{A_i} = \lambda p_i$.

Example 2. Use the principle of thinning (not integrals) to show that, if S and T are independent exponential random variables with parameter λ and μ, then:

$$\mathbb{P}(S < T) = \frac{\lambda}{\lambda + \mu}$$

Example 3. Passengers arrive at a bus stop according to a Poisson process with rate λ. The bus arrives at time $T \sim \mathrm{Exp}(\mu)$, independently of the passengers. Let N be the number of passengers that make it on the bus.

Find the distribution of N using principles of superposition and thinning.