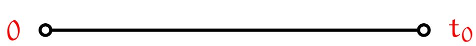
A. **Conditional–Uniformity.** Consider a Poisson arrival process $\{N_t\}_{t \geqslant 0}$ of customers to a shop after t hours, with mean rate λ customers per hour.

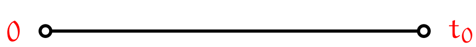
Given that 1 customer arrives in first hour, i.e. given:

$\rightarrow$ find the distribution of that customer's arrival time T in that hour

Conditional–Uniformity. In a Poisson process, **given** that k arrivals have occured by some fixed time t_0, then the distribution of those arrivals are:

For example, suppose we are given that exactly Alice, Bob, and Christian arrive by time t_0. Let's uniformly at random generate arrival times.





Example 1. Customers arrive at a store according to a Poisson process with rate 5 per hour. Suppose that in 2 hours, 8 people have entered the store.

(a) Find the expected time the **last** of these customers entered the store.

(b) Find the probability that 5 of the customers arrived in the first one–and–a–half hours, and the remaining 3 arrived in the last half–hour.

B. **Matrix Multiplication.** To talk about Markov chains, we will need to know how to execute matrix multiplication.

First we consider row times a matrix:

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} =$$

And next, matrix times a matrix:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} =$$