A. **Strong Induction.** Induction is like dominoes falling in a row.

The base case is the flick that knocks down the first domino.

The inductive step is each domino knocking down the next.

The conclusion is that all dominos fall.

It may be useful to make certain changes to the hypotheses.

First, the base case does not have to start at:

Second, the weight of one domino may not be enough to knock down the next.

Perhaps, we need the weight of all preceding dominoes.

That is, a set S may have the following property.

> **strong inductive assumption:** If k is an integer so all integers n from:
>
> then $k+1$ also belongs to S.

The conclusion is the same, all natural numbers after the base case belong to S.

Example 1. The Lucas sequence is the sequence of numbers:

$a_1 = 1, a_2 = 3, a_3 = 4, a_4 = 7, a_5 = 11, a_6 = 18, a_7 = 29, , \dots$

Defined by the rules:

$a_1 = 1$

$a_2 = 3$

$a_n = a_{n-1} + a_{n-2}$ for all $n \geqslant 3$

Let n be a positive integer. Use the principle of strong induction to prove:

The principle of strong induction is also called the second principle of induction.

$a_n < 2^n$

Solution. We use proof by strong induction on n.

Base cases. Our argument will call back two 'dominoes'.

So, we need to establish two base cases: $n = 1$ and $n = 2$.

In these cases the inequality reads:

which are obviously true. ✓

Strong Inductive Step. Let integer k greater than 2 be given.

Assume the inequality holds for all integers n from:

We need to prove the inequality holds for $n = k + 1$, i.e. we need to prove:

To that end we begin from the lefthand side and apply the strong inductive step:

$a_{k+1} =$

Conclusion. The inequality holds for all positive integers n.

□

B. **Binomial Coefficients.** If n is a natural number, and k is a natual number less or equal n, we define the binomial coeffcient n **choose** k by:

$$\binom{n}{k} =$$

For example:

$$\binom{8}{3} =$$

$$\binom{n}{0} =$$

$$\binom{n}{1} =$$

Pascal's Rule. Let n be pos. integer and k be a natural number less than n.

$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$$

Proof. We use the definition of the binomial coefficient:

$$\binom{n}{k} + \binom{n}{k+1} =$$

$\square$

We obtain values of binomial coefficients by constructing **Pascal's triangle**.

0th row:	1
1th row:	1 1
2th row:	1 2 1
3th row:	1 3 3 1
4th row:	1 4 6 4 1
5th row:	1 5 10 10 5 1
6th row:	1 6 15 20 15 6 1
7th row:	1 7 21 35 35 21 7 1

Example 2. Let n be a natural number. Prove that:

$$\binom{2n}{n} = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{n!} \cdot 2^n$$

Solution. We use the definition of the binomial coefficient:

$$\binom{2n}{n} =$$

and then we separate the even and odd terms from the numerator:

$$\cdots =$$

and now concentrate on the even part:

$$\frac{2 \cdot 4 \cdot 6 \cdots (2n)}{n!} =$$